



Elastic instabilities, microstructure transformations, and pattern formations in soft materials

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ABSTRACT

Large deformations of soft materials can give rise to the development of various elastic instabilities. The phenomenon is associated with a sudden and dramatic change in structure morphologies. The underlying mechanism is crucial for the formation of complex morphologies in biology. Moreover, the concept of instability-induced pattern transformations is promising for designing novel materials with switchable functions and properties. In this paper, we review the state of the art in elastic instability phenomena in soft materials. We start by considering the classical buckling in beam-based structure lattice designs. Then, we discuss the instability-induced microstructure transformations in soft porous materials, and heterogeneous multiphase and fiber composites. Next, the mechanisms – often involving the post-buckling consideration – leading to the wrinkling and folding, creasing, fringe, and fingering are discussed.

1. Introduction

Soft materials such as elastomers, gels, and biological tissues can easily develop large deformation in response to various stimuli. The large deformations may lead to the development of elastic instabilities and associated pattern formations. For example, various morphogenesis of flowers and leaves during differential growth [1], the wrinkling of skins on human fingers and toes in response to long immersion in water [2,3], and the creasing in dough during the constrained swelling [4] have been observed. The elastic instabilities are usually accompanied by a dramatic change in structure configurations and a sudden loss in load capacities. Historically, the phenomenon is perceived as a failure mode (for example, buckling of columns and shells). Recently, the new concept of using the rich instability-induced pattern transformations in soft materials has been put forward. Examples include mechanical metamaterials with unusual properties [5], photonic [6] and phononic [7,8] switches, soft robot [9,10], sensors [11], flexible electronics [12], adhesion [13] and energy absorption [14]. Moreover, the knowledge about the mechanical instability phenomenon can help elucidate the morphogenesis in organs during growth in various biological systems [15–17].

The diverse potential applications of the instability phenomena in soft materials motivated a significant body of theoretical, numerical,

and experimental studies. These works addressed the rich variety of elastic instabilities, including buckling, wrinkling, folding, creasing, cavitation, fringe, and fingering in various soft material systems under different stimuli [18–23]. In this paper, however, we will focus on the instability phenomena induced by *mechanical* loadings. The paper is structured as follows. Section 2 presents recent advances in the buckling of beam-like structures, including single beam, fiber composite, porous composite, and lattice structure. Section 3 discusses the wrinkling of stiff films on a compliant flat soft substrate. Section 4 reviews various instabilities emerging in a constrained soft layer. The review is concluded by a summary and discussion.

2. From beams and fibers to fiber composites, periodic porous and lattice structures.

2.1. Single beam

The study of elastic instability dates back to the 18th century when Leonard Euler introduced the calculation of the critical load for the buckling of a slender beam in 1744 [24], followed by Lagrange's analysis for higher modes in 1770 [25] (Fig. 1(a)). The critical buckling load F_c of a homogeneous linear elastic slender beam with hinged ends can be calculated as

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$$F_c = \frac{\pi^2 EI}{L^2}, \quad (1)$$

where E is Young's modulus, I is the second moment of area, L is the beam length. Recently, Coulais et al. [26] pointed out that the slope of the force–displacement curve of Euler's slender beam in the post-buckling regime is $1/2$ (Fig. 1(b)). Moreover, and the slope is independent of the end boundary conditions. However, for the non-linear elastic beams with larger aspect ratios ($t/L > 0.12$, t refers to the beam width), the material nonlinearities can lead to the discontinuous buckling. This postbuckling regime is characterized by a decrease in the compressive force decreases with an increase in the applied deformation (Fig. 1(b)). In beams with larger aspect ratios ($t/L > 0.24$), Chen and Jin [27,28] identified a new snapping-back mode. In this scenario, both force and displacement decrease after the onset of buckling. The authors show that for $t/L > 1$, the beam transitions from a snapping-back mode to the barreling mode. Remarkably, the phase diagram can be violated with specially designed metabeams. For example, Coulais et al. [26] indicated that a slender meta-beam consisting of periodic voids in a soft matrix exhibits discontinuous buckling; Oliveri and Overvelde [29] utilize the topology optimization of beam geometry to design meta-beams with targeted buckling load or maximum buckling load.

The beam buckling behavior significantly depends on the type of loadings and boundary conditions. Lazarus et al. [30] show the wavy-like buckling of a slender rod under twisting deformation and analyzed the role of the initial rod curvature on the buckling behavior (Fig. 1(c)). Miller et al. [31,33] investigate the buckling of a thin elastic rod in a cylindrical constraint, identified the sequential buckling from straight to sinusoidal, then to helical shape configurations, and, finally, to the lock-up state due to the friction (Fig. 1(d)). For the frictionless

case, Xiao et al. [34] show that the helical configuration transitions to an alpha shape.

Another important situation arises when the beam is surrounded by a softer matrix. Such constraint results in a more stable behavior (as compared to the isolated beam buckling), and the buckling behavior is dictated by the contrast in matrix-to-beam material properties (in addition to the boundary conditions). Su et al. [35] show that the occurrence of the planar wavy pattern and non-planar coiled buckling mode of stiff fiber surrounded by a soft matrix depends on the interplay of the two lowest buckling modes. Zhao et al. [36] examined the buckling of short elastic fiber in a soft matrix; their theoretical analysis shows that the buckling of fibers can be tuned by several orders of magnitude via altering the length ratio of stiff fiber over the height of the soft matrix. Chen et al. [37] numerically illustrate that the long stiff wire embedded in a soft matrix initially buckles in 2D sinusoidal configuration, and then gradually transits from the 2D sinusoidal into the 3D helical mode. Li et al. [32] experimentally investigated the buckling of a 3D printed fiber embedded in a soft matrix (Fig. 1(e)). The work introduced an explicit formula for the buckling wavelength estimation,

$$\log\left(\frac{2\pi d}{L_{cr}}\right) = -0.265\log\left(\frac{G_f}{G_m}\right) + 0.265\log\left(\frac{4}{1 + \nu_f}\right), \quad (2)$$

where d and L_{cr} are the fiber diameter and critical wavelength, respectively; G_f and G_m are the shear moduli of stiff fiber and soft matrix materials; ν_f is the Poisson's ratio of stiff fiber material; here the soft matrix is assumed to be incompressible, i.e., $\nu_m = 0.5$. Note that Eq. (2) is obtained under the assumption of negligible shear deformation in the fiber and matrix. While this assumption provides a good approximation for large fiber-to-matrix modulus ratios, the shear deformation is

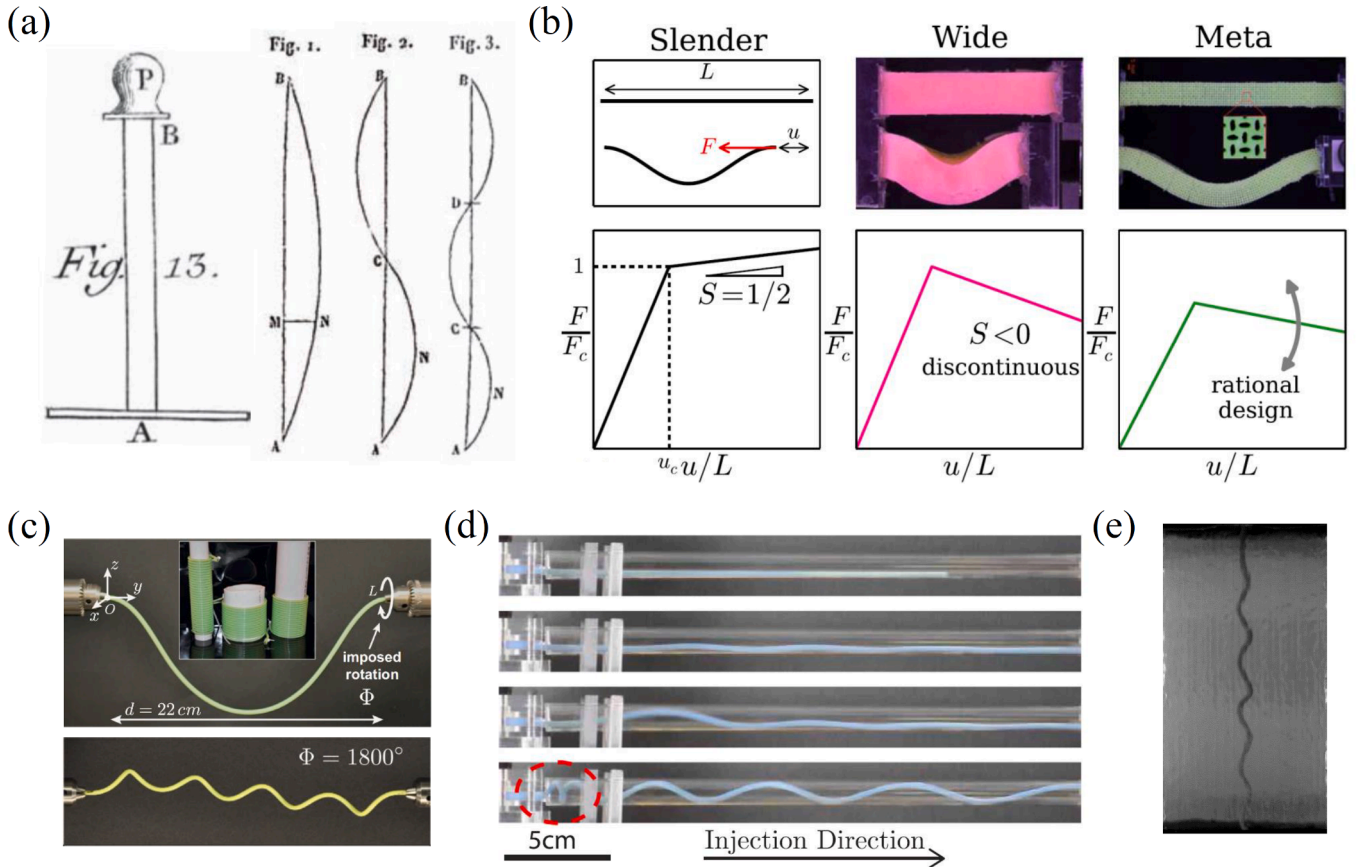


Fig. 1. Single-beam buckling. (a) Classical Euler buckling [24,25]. (b) Hyperelastic slender-, wide- and meta-beam, adapted from Ref. [26]. (c) Beam buckling under twisting deformation, adapted from Ref. [30]. (d) Buckling of beam confined in a hollow cylinder, adapted from Ref. [31]. (e) Buckling of beam embedded in a soft matrix, reproduced with permission from Ref. [32].

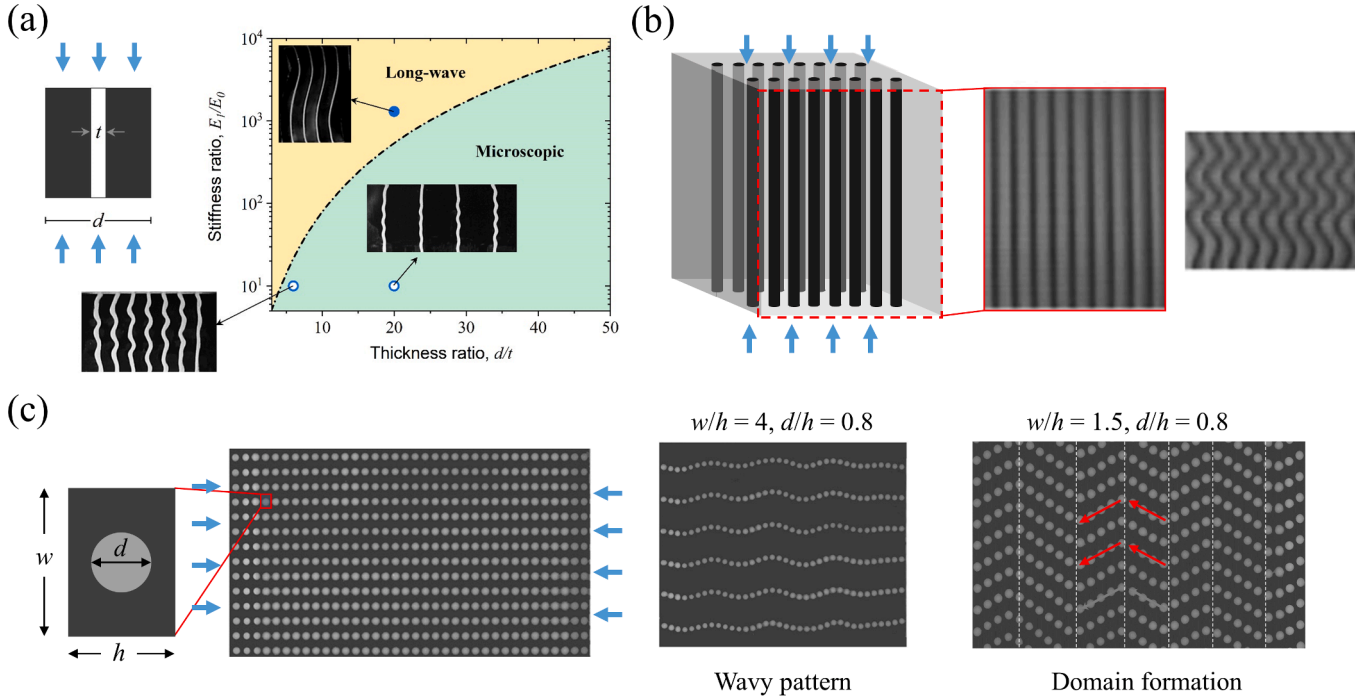


Fig. 2. Instabilities and pattern formations in periodic heterogeneous soft composites. (a) Layered composites, reproduced with permission from Ref. [59]. (b) 3D fiber composites, reproduced with permission from Ref. [32]. (c) Particulate composites, reproduced with permission from Ref. [62].

essential for the case with a small modulus ratio [38]. Thus, the estimate provides more accurate predictions for relatively stiff fibers. In particular, Li et al. [32] reported a good agreement between the prediction of Eq. (2) and finite element simulations in the range of $G_f/G_m > 100$.

2.2. Fiber composite

Fig. 2 illustrates the buckled patterns in periodic stiff-fiber-soft-matrix composites. Due to the interaction between neighboring fibers, the buckling behavior of the fibers is determined by their volume fraction and material properties. Rosen [39] analyzed the stability of linear elastic layered composite, for which the buckling strain is

$$\epsilon_{cr} = \frac{G_m}{E_f} \frac{1}{c_f(1 - c_f)} \quad (3)$$

where c is volume fraction, the properties of the fiber and matrix are denoted by subscript f and m , respectively. By considering the interacting stress between matrix and fiber interface, Parnes and Chiskis [40] re-examined Rosen's solution and showed that the buckling of layered composite has two separate regimes. In particular, the dilute composites buckle at finite wavelengths, while the non-dilute composites buckle at infinite large wavelengths or so-called long-wave mode; the buckling strain of the non-dilute composites agrees with the Rosen's solution [39].

In the instability analysis within the non-linear elasticity framework, these modes are frequently referred to as microscopic and macroscopic instabilities. The instability analysis is performed in terms of the linearized incremental equations [41] for small motions superimposed on the finite deformation state. The detection of microscopic instabilities relies on the Bloch-Floquet analysis applied for periodic composites [42]. Geymonat et al. [43] showed the equivalence for the long-wave limit in the Bloch-Floquet analysis and the loss of ellipticity condition. The loss of ellipticity condition is frequently utilized to predict the onset of the macroscopic (or long-wave) instabilities. The analysis requires the determination of the tensor of the elastic moduli (or related acoustic tensor), which can be deduced from the constitutive laws in terms of the

energy density functions. By employing the phenomenologically-based energy functions, the stability of fiber-reinforced hyperelastic composites has been derived [44–47]. Recently, Hamdaoui et al. [48] used a similar approach to predict the fiber kinking and splitting failure modes in aligned fiber-reinforced hyperelastic solids. Moreover, Demirkoparan et al. [49,50] utilized the phenomenological models to study the effect of pre-deformation and anisotropy on the bulging and helical buckling behavior of soft tubes. An alternative approach is based on the effective or homogenized responses of the composites utilizing the micro-mechanics consideration. For example, Agoras et al. [51], Rudykh and Debotton [52] used the method to predict the onset of macroscopic instabilities in transversely isotropic fiber composites with hyperelastic phases. Employing the numerical Bloch-Floquet analysis, Li et al. [53] studied the elastic instability in compressible laminate and found that the phase compressibility has the stabilization effect. Slesarenko and Rudykh [54] identified both the macroscopic and microscopic instabilities in 3D periodic hyperelastic fiber composites loaded along the fiber direction. Galich et al. [55] studied the influence of 3D fiber arrangement on the instabilities showing, for example, that the composite with a higher in-plane periodicity aspect ratio is more prone to instabilities. Recently, Arora et al. [56] examined instabilities in 3D fiber composites with non-Gaussian hyperelastic phases illustrating the significant influence of the phase stiffening on the composite stability. Greco et al. [57,58] investigated the influence of matrix or fiber/matrix interface microcracks on the failure behaviors of periodic fiber-reinforced composites.

Recent developments in material fabrication and 3D-printing of soft multiphase composites allowed the experimental realization of the instability phenomenon in soft composites. For example, Li et al. [59] reported the experimental observation of wrinkling in interfacial layers subjected to in-plane compressive deformation (Fig. 2(a)). Slesarenko and Rudykh [60] experimentally illustrated the tunability of wrinkling patterns in soft layered composites via viscoelasticity. Arora et al. [61] studied the role of interphase layers on the instabilities of laminate composites. Li et al. [32] investigated elastic instabilities and pattern formations in 3D-printed deformable fiber composites and observed the transition of the instability induced patterns from small wavelength

wavy patterns to the long-wave mode. Galich et al. [55] numerically predicted that the co-operative buckling of the fiber could happen in the “stiff” direction where the fibers are close to each other. This interesting buckling behavior has been observed in recent experiments on 3D-printed periodic fiber composites (Fig. 2(b)).

Particulate composites are another important class of soft heterogeneous materials exhibiting the elastic instability phenomenon that can be used to switch the functions and properties through pre-designed microstructure transformations. The composites with the random distribution of *circular* particles are stable [63]; however, similar random distribution composites with *aligned stiff elliptical* inclusions exhibit the macroscopic instability; this difference is due to the symmetry breakage

in the elliptical fiber orientation [64]. For periodic composites, Michel et al. [63] and Triantafyllidis et al. [65] numerically predicted the onset of microscopic and macroscopic instabilities in the two-phase hyperelastic solids under biaxial loadings.

Recently, Li et al. [62] experimentally observed the pattern transitions in periodic circular stiff inclusion composites (see Fig. 2(c)). The wavy-like buckling patterns are reported in the composite with sparse inclusions, whereas the anti-symmetric domains emerge in the composite with a dense inclusion arrangement. Note that the wavelength of the experimentally observed domains is comparable to the characteristic size of composite microstructure, although the numerical instability analysis predicts the long-wave mode. Similar domain or twinning

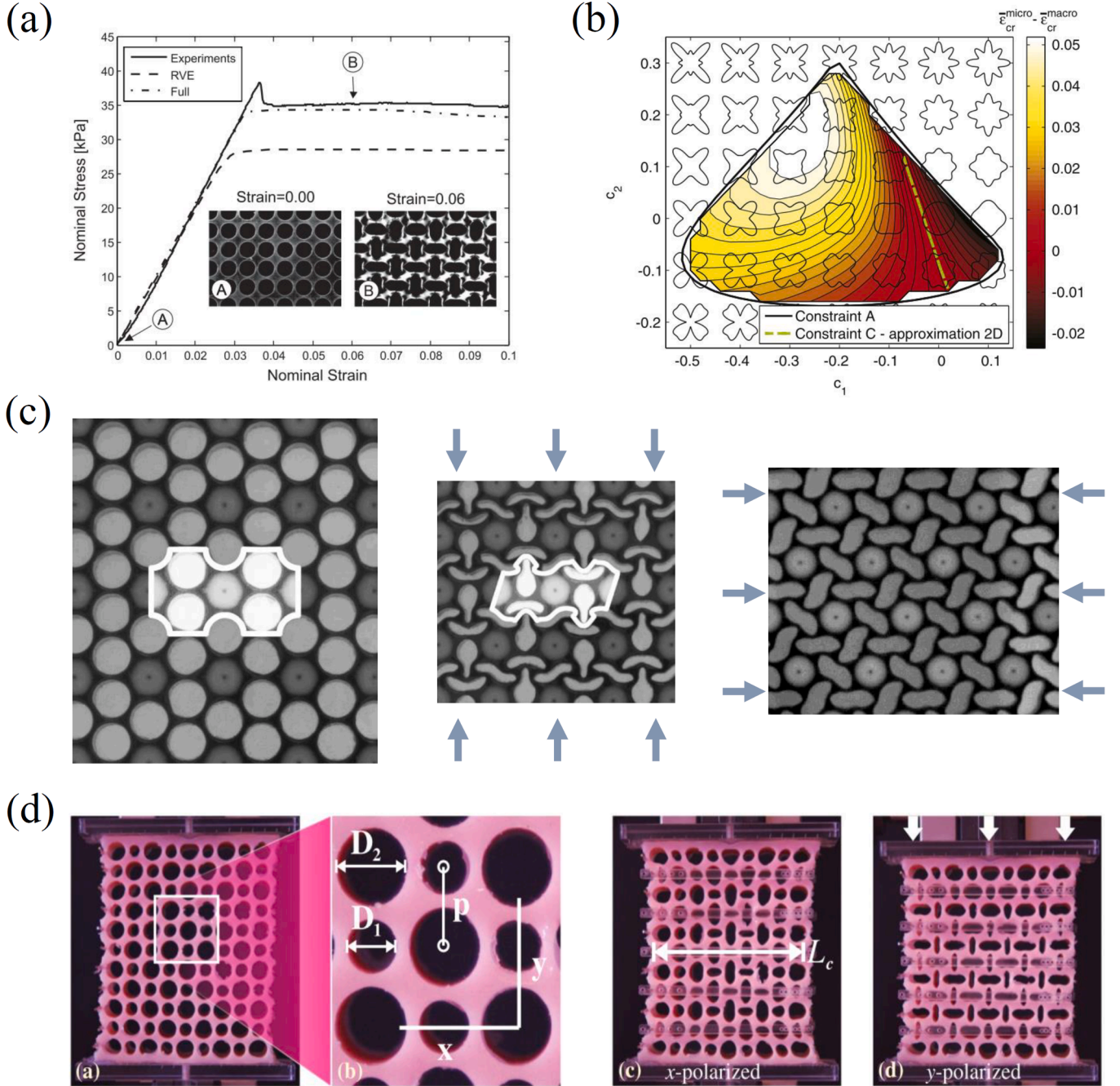


Fig. 3. Instability-induced pattern transformations in periodic porous composites. (a) Stress-strain curve for a square array of circular voids embedded in an elastomer with associated undeformed and buckled configurations, adapted from Ref. [87]. (b) Difference in critical strain between microscopic and macroscopic instabilities as functions of geometrical coefficients, adapted from Ref. [92]. (c) Multiple pattern formations in void-inclusion-matrix composites, reproduced with permission from Refs. [96,97]. (d) Programmable bi-sized void system through external constraints, adapted from Ref. [99].

patterns were also identified in other material systems, such as thin films on compliant substrates [66–68], liquid crystals [69], and nematic elastomers [70,71].

It is worth mentioning that the heterogeneous composites can also be utilized in electro/magneto-active elastomeric composite design to enhance their pattern morphologies and actuation capability [72]. The instabilities in these composites developing under large deformations in external (electric or magnetic) fields are a crucial issue for the material design and application [73]. While this review paper is focused on the material instabilities under pure mechanical loadings, interested readers are referred to recent relevant works on these topics [74–80].

2.3. Porous composite

The instability phenomenon in soft porous materials manifests in the sudden collapse of voids upon reaching the critical deformation level (see Fig. 3). For porous composite with a *random* distribution of circular voids, only macroscopic instabilities are detected by the homogenization method [81–83], predicting the development of the shear band-type instability in the post-buckling regime [81,82]. For periodic composites, however, both microscopic and macroscopic instabilities with associated pattern transformations are predicted theoretically and numerically, and realized experimentally [65,83–90]. Interestingly, the periodically distributed circular voids suddenly switch into ellipses arranged in the mutually orthogonal configuration due to the elastic instability phenomena [87] (Fig. 3a). The underlying mechanism of the pattern transformations in porous composites is the buckling of thin ligaments between voids, as well as the corresponding folding mechanisms. The initial void shape plays a significant role in the instability induced pattern transformations [91,92] (Fig. 3b); thus, the topological optimization can help achieve the targeted functionalities [92,93]. To rationalize the folding mechanisms in the periodic porous composites, a simplified model has been introduced. In particular, the holey sheet is considered as a network consisting of the corner-rigid-hinged polygons that can rotate freely at the hinges [7]. Constrained by the geometrical compatibility, only limited folding patterns are allowed [94,95]. Remarkably, multiple folding patterns can be achieved in the same porous composites by altering the loading directions [7]. Shim et al. [95] further utilized the folding patterns in designing 3D Buckliball. Through the numerical simulation and experiments, the guidelines for the pattern design of the Buckliball have been provided. Thus, the Buckliball behavior can be pre-designed by the geometry of thin shell ligaments.

Recently, Li et al. [96,97] put forward the concept of multiphase composites consisting of periodic stiff inclusions and voids distributed in a soft matrix. The proposed composites exhibit multiple pattern transformations through the positioning of stiff inclusions and composite anisotropy (Fig. 3c). The system exhibited wide tunability of the Poisson's ratio, attaining the extreme negative levels of auxetic behavior. In addition, the application of the system to control elastic wave propagation and band gaps has been demonstrated numerically [97,98].

An interesting concept of programming the void-matrix system via lateral confinement has been proposed [99,100] to tailor the force–displacement response. In particular, the bi-sized void system (illustrated in Fig. 3d) showed monotonic, non-monotonic, and hysteretic behavior depending on the lateral confinement strain [99,100]. In addition to the rich pattern transformations of periodic porous composites, Overvelde et al. [92,101] discovered that the elastomer with a square array of circular pores under equibiaxial deformation develops tensile instability, resulting in a checkerboard pattern with two different pore sizes.

2.4. Lattice structure

While the beam elements of lattice structures can experience local failure described by the classical Euler buckling, the overall structural

co-operative behavior is rather different and opens potential ways for utilizing the instability phenomenon [102,103]. For example, a tilted elastic beam-element can snap between different stable configurations [104], even further, retain the buckling state after unloading [14] (Fig. 4(a)). This mechanism can be used to trap the elastic energy and, thus, to design energy absorption metamaterials. A similar mechanism was employed in the double-curved beams under tensile load [105]. A different design of lattice structures with varying beam thicknesses was proposed to design fault-tolerance materials [106]. Through the pre-designed sequence of localized buckling and evenly distributed failure of thin and thicker beams, the fault-tolerant lattice material showed the energy absorption capacity (Fig. 4(c) and (d)) [106]. Fernandes et al. further illustrated the improvement of buckling resistance of lattice composite through optimizing the arrangement and thickness of beams [107]. To overcome the limitation of deformation sequence uncertainties (induced by the imperfections stemming from, for example, manufacturing processes and/or boundary conditions) in metamaterials consisting of identical unit cells, a pre-designed variation in beam thickness can be utilized to obtain a deterministic deformation sequence [108].

2D lattice structures subjected to biaxial loadings show multiple buckled patterns that can be tuned by varying the combination of loadings in different directions and lattice microstructure. By applying the beam-column matrix method, Haghighpanah et al. [109] systematically examined lattice structures' buckling. The authors produced the buckling maps for biaxially loaded lattices with square (Fig. 4b), triangle, and hexagonal grids. Notably, different buckling patterns can be switched by altering the cross-section of the beams [112]. Furthermore, with the advancement in material fabrications, recent works showed that the buckling pattern of lattice structure could be induced not only by mechanical loading [113], but also by temperature [114], electrical field [115], and hydration-induced swelling [116,117].

The snapping of a single beam under confined geometrical constraints can be populated into 3D structures. For example, polymeric 3D micro-lattice shows highly repeatable mechanical energy-absorbing through allowing the tailored buckling mechanism (Fig. 4e); self-recovering and non-self-recovering lattices can be pre-designed through altering lattice geometry [110,118]. Mechanical metamaterial design with a strategical assembling of cubic building blocks consisting of beams with outward and inward buckling have been proposed to achieve programmable shape changes; the design has been illustrated by the flat metacube programmed into a smile face-like shape changes under uniaxial compression (Fig. 4f) [111].

3. Stiff film on a compliant flat soft substrate

When the system of a thin stiff film on a compliant flat soft substrate is uniaxially compressed beyond a critical level, wrinkling instability develops (as illustrated in Fig. 5(a)). Since the bending energy and stretching energy scale as (see for example, [119])

$$U_{\text{bending}} \sim Et^3 \int (1 - \nu^2) \int \kappa^2 dV, \quad \text{and} \quad U_{\text{stretching}} \sim Et \int (1 - \nu^2) \int \epsilon^2 dV \quad (4)$$

Here, t is the stiff film thickness, κ is the bending curvature. For small t , $U_{\text{bending}} \ll U_{\text{stretching}}$, thus, the thin film prefers the bending mode with long-wavelength; the buckling of the underlying soft substrate, however, favors short wavelengths [120,121]. Based on the energy consideration of the bending and stretching modes, one can conclude that the interaction between stiff film and soft substrate results in a characteristic length of wrinkling pattern. Through considering the thin film as an elastic non-linear von Karman beam and the soft substrate as a semi-infinite solid, the critical wrinkling strain ϵ_{cr} and wrinkling wavelength λ_{cr} in small deformation (usually defined as the deformation level is less than 5%) can be calculated by [66,122]

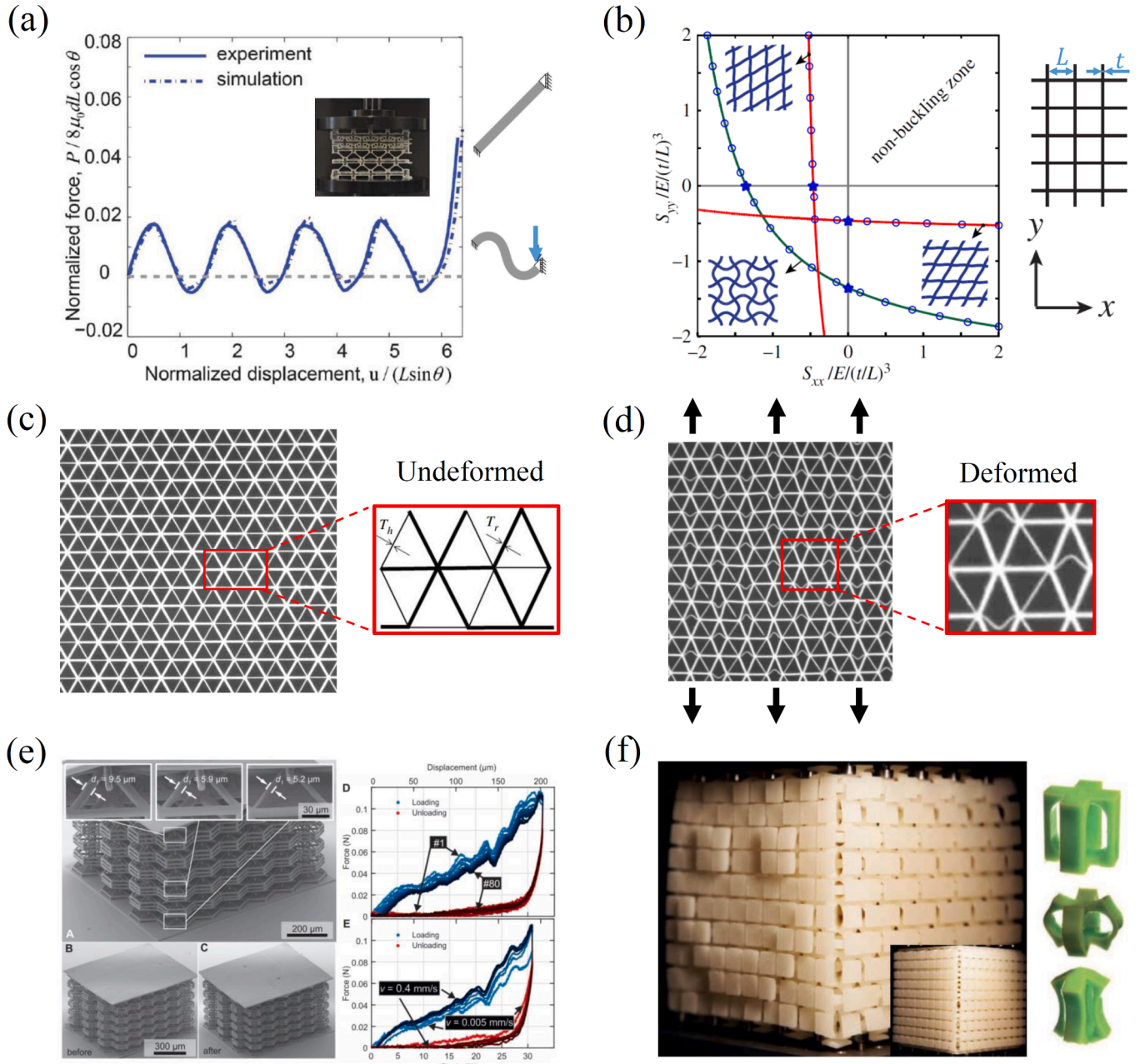


Fig. 4. Buckling and functionalities of lattice composites. (a) Force-displacement response of an elastic multistable structure consisting of tilted elastic beams, adapted from Ref. [14]. (b) Buckling map of 2D lattice structure with a square grid, adapted from Ref. [109]. (c) Fault-tolerant lattice with varying thickness in undeformed (c) and deformed (d) states, reproduced with permission from Ref. [106]. (e) Pictures of microlattice before and after applied deformation, and corresponding force-displacement measurements, adapted from Ref. [110]. (f) Shape-morphing metamaterials consisting of programmed building blocks with inward and outward buckling patterns, adapted from Ref. [111].

$$e_{cr} = \frac{1}{4} \left(\frac{3\bar{E}_s}{\bar{E}_f} \right)^{\frac{1}{3}}, \quad \text{and} \quad \lambda_{cr} = 2\pi t \left(\frac{\bar{E}_f}{3\bar{E}_s} \right)^{\frac{1}{3}} \quad (5)$$

where $\bar{E} = E/(1 - \nu^2)$ is the plane-strain modulus. The predictions of Eq. (5) are in good agreement with experimental results for the system with large modulus contrast [123]; these systems buckle at relatively small strain levels. However, for lower modulus contrasts, large deformations are required to trigger the buckling. In this case, the experiments showed that the wrinkling wavelength depends on the applied pre-strain in the substrate [124,125]. This observation was explained by a refined mechanical model considering the geometrical and material nonlinearities

under large deformation [124,126–128].

The instability-induced wrinkling may transit to more complex patterns with further deformation. These secondary patterns include folds [129–131], ridge [132,133], period-double [134–136], period-quadrupling [134], and hierarchical wrinkles [137,138]; alternative scenarios include surface failure through cracks [139] or delamination [140–142]. In particular, for the system with a moderate modulus contrast between stiff film and soft substrate, the stiff film wrinkles may transit to crease [132,143], or even directly developing creasing instability [144]; the latter is the homogeneous material buckling form that will be discussed in Section 4. Through setting adhesion energy between film and substrate, Wang and Zhao [145–147] produced a phase map to describe the pattern selections in the system with different modulus

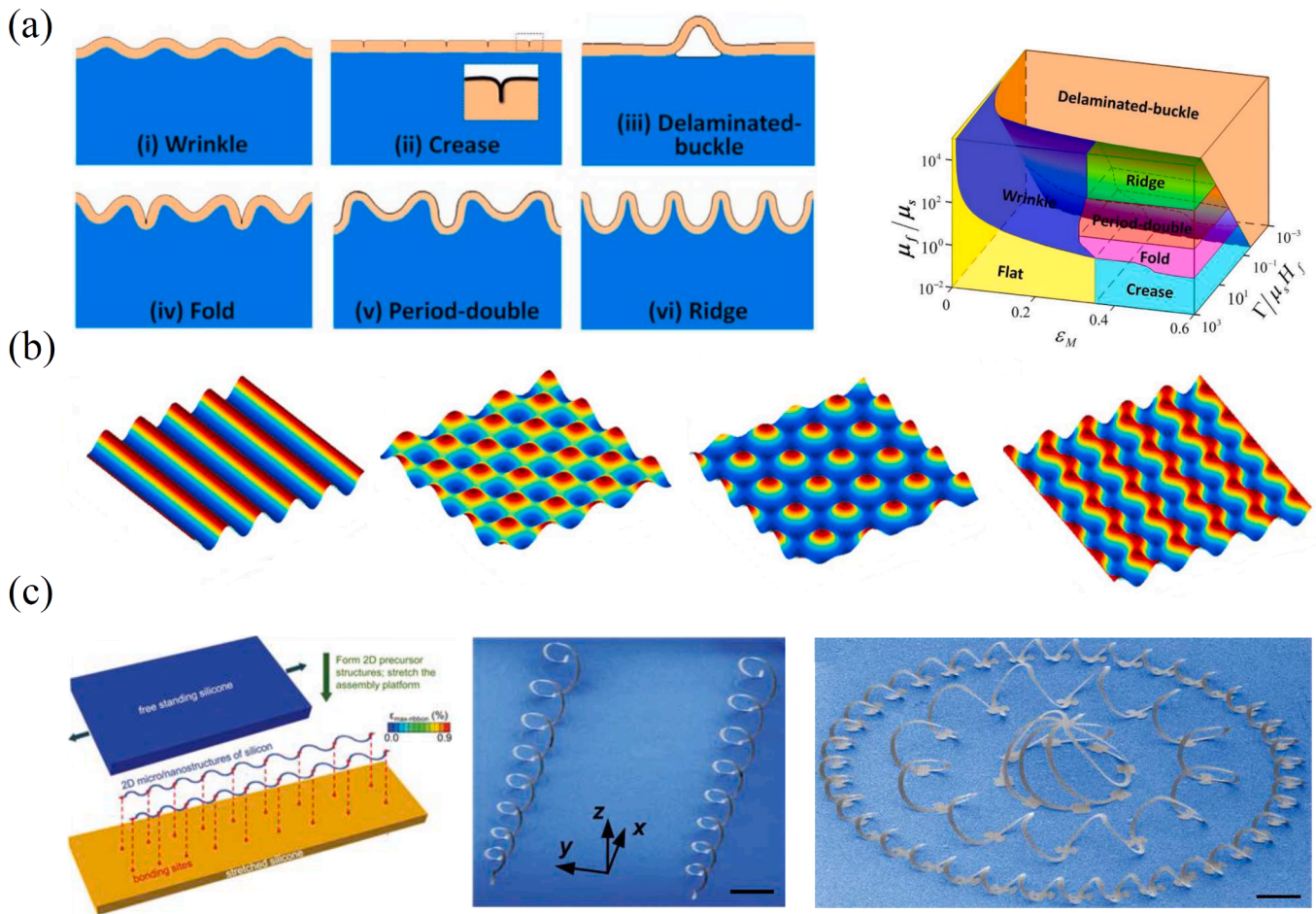


Fig. 5. Pattern selections of stiff films on a compliant soft substrate. (a) Under uniaxial deformation, adapted from Ref. [145]. (b) Under biaxial deformation, adapted from Ref. [68]. (c) Buckling of partially bonded stiff films on a compliant soft substrate, adapted from Ref. [163].

contrasts, mismatch strains, and adhesion energy (Fig. 5(a)).

The stiff film-soft substrate system produces a rather different pattern when subjected to biaxial compressive deformation. The stiff film can develop wavy mode, ridge, square checkerboard mode, hexagonal mode, triangular mode, herringbone mode, and labyrinths patterns (Fig. 5b) [66,68,122,148–154]. It has been shown that the pattern selections and transitions significantly depend on the initial imperfection, loading history, and applied deformation level [68,155–157]. For equibiaxial loading, the herringbone pattern usually has the minimum energy state [66,68,122,158]. Notably, the herringbone pattern has the minimum buckling strain at infinite large longitudinal wavelength [67], while experimental results showed that the longitudinal and transverse wavelengths of herringbone patterns are comparable [149,159]. Audoly and Boudaoud [155,156] discussed this discrepancy and indicated that this could be because the system is trapped in a local minimum energy state instead of developing the buckling mode for a global minimum one due to the loading history. Through a sequential loading, Lin and Yang [154] experimentally observed a well-ordered herringbone mode. The sequential loading induced a minimum energy configuration at a finite longitudinal wavelength [148]. It is worth noting that similar pattern transitions were also observed in the system of stiff films attached on a curved substrate. In these systems, the patterns are strongly regulated by the structure curvature; interested readers are referred to the recent review focusing on pattern formations in curved substrates [160]. The buckling of a stiff film partially bonded to a pre-stretched elastomer exhibits even more complex patterns [161,162]. Through the pre-designed bonding point and pre-designed shape of the stiff film, complex 3D patterns can be achieved in the out-of-plane deformation of the

film (Fig. 5c) [163–166]. For more details of the underlying mechanisms, the readers are referred to the relevant reviews [167,168]. Note that the buckling of the partially bonded stiff-film-soft-substrate system has been widely utilized to fabricate complex 3D mesostructures for potential applications in flexible electronics [161,169].

4. Soft layer confined by a stiff substrate

When a soft material confined by a stiff substrate is compressed beyond a critical level, the flat surface develops the so-called creases – the patterns with highly localized self-contact folds (Fig. 6a). The creases have been observed in experiments involving compressive deformation of an elastomer or hydrogel [4,170–175], volumetric expansion or shrinkage of swollen hydrogel or gel [176–179], mass addition of tissue growth [180–182], even dielectric elastomer subjected to external electric field [183,184]. Gent and Cho [170] experimentally observed the creasing in a bent rubber block surface at a compressive strain of around 0.35; this value is much smaller than Biot's [120] theoretical prediction of the critical strain 0.457 for the wrinkling of a semi-infinite neo-Hookean material under compressive plain-strain conditions.

To explain this discrepancy and to provide the understanding of the creasing mechanisms, finite element simulations have been utilized. The numerical results – for neo-Hookean material under plane-strain conditions – predict that the free flat surface undergoes a discontinuous transition from a smooth surface to a localized crease pattern at critical strain about 0.35 [171,185]; this result is an excellent agreement with Gent and Cho's [170] experiments. Further numerical investigations showed that the crease sets a lower energy state than the homogeneous

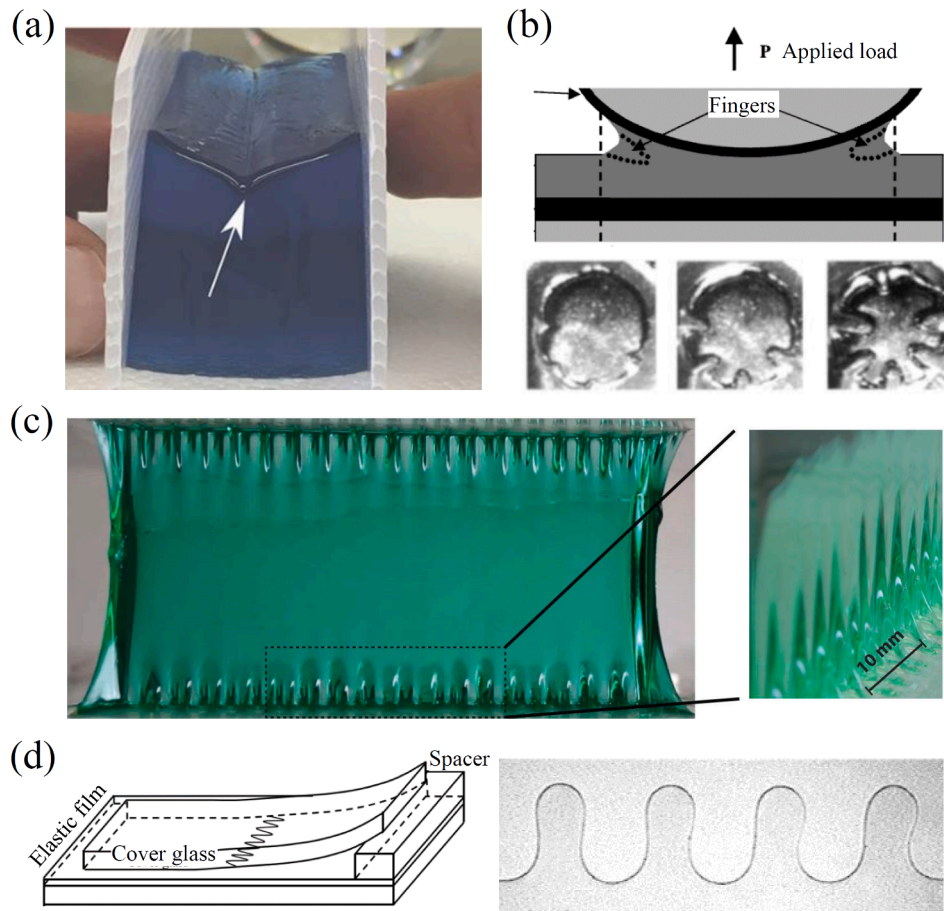


Fig. 6. Bucking of a soft layer confined by a stiff substrate. (a) Creasing instability, adapted from Ref. [174], (b) Fingering instability, adapted from Ref. [195], (c) Fringe instability, adapted from Ref. [196], (d) Interfacial instability, adapted from Ref. [197]. Note that the fingering instability undulates in the middle of the soft layer, while the fringe instability undulates at the edge of the soft layer.

state [171,186]. Cao and Hutchinson [121] showed that the wrinkling in homogeneous neo-Hookean materials is extremely unstable and dynamically collapse to creases. Hohlfeld and Mahadevan [187] pointed out that the crease is a new instability mode different from the wrinkling of a stiff film on a compliant soft substrate. While a linear perturbation analysis can accurately predict wrinkling, the identification of creasing requires the full finite deformation solutions. Tang et al. [188] observed the dependence of creasing length on the specimen geometry in experiments. Moreover, the crease instability in the systems with graded or pre-deformed bilayers can be supercritical and subcritical [189,190]; this depends on the mismatch deformation [175] or modulus/thickness ratio [144]. The crease formation is also significantly affected by material properties. For example, the material strain-stiffening [191], plasticity [192], or inelasticity [193] can delay or even eliminate the onset of creasing. By applying a singular perturbation analysis, Ciarletta [174,194] derived an asymptotic approximation of the creasing strain, which was also validated by numerical simulations.

In addition to the crease instabilities in the confined soft layer subjected to compressive deformation, the tensile deformation can also induce various instability modes. For example, cavitation, fingering (Fig. 6b), and fringe instabilities (Fig. 6c) are observed in soft layers with perfect adhesion to a stiff substrate under a tensile deformation [195,196]. The cavitation instability refers to a sudden growth of voids in soft elastomer that occurs at microscopic length-scales under a large hydrostatic tensile stress [198,199]. This crucial mechanism for fracture failure of soft materials has been extensively investigated – interested readers are referred to the focused reviews on cavitation [200–202]. Differently, fingering and fringe are usually observed at macroscopic

length-scales and exhibit periodic fingers of air that invade the elastic layers due to the material incompressibility and applied boundary constraints [196,203,204]. In particular, the fingering instability is associated with a monotonic load–displacement curve, while the load–displacement curve of the soft layer with fringe instability is non-monotonic [196,205]. The selection of these instability modes depends on the shape and geometrical parameter material property of the soft layer [196,206]. Upon onset of instability, localized large deformations develop, and the material strain-stiffening can delay the emergence of fringe and fingering instabilities [207]. This mechanism is similar to the crease smoothing by material strain-stiffening [191].

Finally, consider the possible scenario of the interfacial fracture of the adhesion between the soft layer and the stiff substrate (see Fig. 6d). In this case, the delaminated surface of the soft layer can develop periodic undulations, giving rise to the fingering instabilities [208–213]. This fingering instability is an analog of the Saffman–Taylor instability observed in the fluid injected into another fluid with higher viscosity. Although both instabilities share similar fingering patterns, however, the characteristics of the formation of these patterns are fundamentally different. The fingering instability in soft solids is reversible and subcritical, and the undulation wavelength depends on the soft layer thickness [197,204,214]. In contrast, the Saffman–Taylor instability is irreversible and supercritical, and the wavelength of the fingering pattern depends on the viscosity and surface tension [215].

5. Concluding remarks

The review summarizes the development of the theoretical

prediction, numerical modeling, and experimental observation of the elastic instability phenomenon and subsequent post-buckling behavior. We note that the elastic instability phenomenon is rather sensitive to the structure geometry and material imperfections. These imperfections can significantly affect the initiation of buckling modes and their development into post-buckling pattern transformations. For example, from a numerical modeling perspective – where the finite element method is frequently used – the imperfections are introduced into the idealized system in a pre-defined manner. In experiments, the imperfections can stem from a variety of factors ranging from boundary effects of finite size samples, breakage in the microstructure periodicity, loading alignments and friction, to defects and geometrical imperfections, and variations in local material properties. Moreover, soft materials exhibit their essential inelastic behavior. The current theoretical and numerical modeling of instabilities in soft materials mostly focused on the elastic systems, and significant efforts are required to generalize the framework for inelastic systems. Furthermore, the application of the instability mechanisms for elucidating the morphogenesis of tissues and organs require the consideration of so-called coupled multiphysics problems, and the development of constitutive models capable of capturing the high complexity of living biological materials frequently developing residual stresses [216,217]. In particular, the coupling of chemical, biological, and mechanical factors with complex interactions between different fields across length-scales in living materials presents a challenge for the classical continuum mechanics approaches [218,219]. This is even more so for the development of buckling and post-buckling analysis and modeling. The process may be facilitated by incorporating the promising approaches of machine learning.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] C. Huang, Z. Wang, D. Quinn, S. Suresh, K.J. Hsia, Differential growth and shape formation in plant organs, *Proc. Natl. Acad. Sci.* 115 (2018) 12359–12364, <https://doi.org/10.1073/pnas.1811296115>.
- [2] K. Kareklas, D. Nettle, T.V. Smulders, Water-induced finger wrinkles improve handling of wet objects, *Biol. Lett.* 9 (2013) 20120999, <https://doi.org/10.1098/rsbl.2012.0999>.
- [3] E.P.V. Wilder-Smith, Water immersion wrinkling: Physiology and use as an indicator of sympathetic function, *Clin. Auton. Res.* 14 (2004) 125–131, <https://doi.org/10.1007/s10286-004-0172-4>.
- [4] S. Cai, K. Bertoldi, H. Wang, Z. Suo, Osmotic collapse of a void in an elastomer: Breathing, buckling and creasing, *Soft Matter* 6 (2010) 5770–5777, <https://doi.org/10.1039/c0sm00451k>.
- [5] D.M. Kochmann, K. Bertoldi, Exploiting microstructural instabilities in solids and structures: from metamaterials to structural transitions, *Appl. Mech. Rev.* 69 (2017) 050801, <https://doi.org/10.1115/1.4037966>.
- [6] D. Krishnan, H.T. Johnson, Optical properties of two-dimensional polymer photonic crystals after deformation-induced pattern transformations, *J. Mech. Phys. Solids* 57 (2009) 1500–1513, <https://doi.org/10.1016/j.jmps.2009.05.012>.
- [7] S. Shan, S.H. Kang, P. Wang, C. Qu, S. Shian, E.R. Chen, K. Bertoldi, Harnessing multiple folding mechanisms in soft periodic structures for tunable control of elastic waves, *Adv. Funct. Mater.* 24 (2014) 4935–4942, <https://doi.org/10.1002/adfm.201400665>.
- [8] P. Wang, F. Casadei, S. Shan, J.C. Weaver, K. Bertoldi, Harnessing buckling to design tunable locally resonant acoustic metamaterials, *Phys. Rev. Lett.* 113 (2014) 014301, <https://doi.org/10.1103/PhysRevLett.113.014301>.
- [9] D. Yang, B. Mosadegh, A. Ainla, B. Lee, F. Khoshdel, Z. Suo, K. Bertoldi, G. M. Whitesides, Buckling of elastomeric beams enables actuation of soft machines, *Adv. Mater.* 27 (2015) 6323–6327, <https://doi.org/10.1002/adma.201503188>.
- [10] T. Chen, O.R. Bilal, K. Shea, C. Daraio, Harnessing bistability for directional propulsion of soft, untethered robots, *Proc. Natl. Acad. Sci.* 115 (2018) 5698–5702, <https://doi.org/10.1073/pnas.1800386115>.
- [11] J.Y. Chung, A.J. Nolte, C.M. Stafford, Surface wrinkling: A versatile platform for measuring thin-film properties, *Adv. Mater.* 23 (2011) 349–368, <https://doi.org/10.1002/adma.201001759>.
- [12] X. Cheng, Y. Zhang, Micro/nanoscale 3D assembly by rolling, folding, curving, and buckling approaches, *Adv. Mater.* 31 (2019) 1–27, <https://doi.org/10.1002/adma.201901895>.
- [13] E.P. Chan, E.J. Smith, R.C. Hayward, A.J. Crosby, Surface Wrinkles For Smart Adhesion, *Adv. Mater.* 20 (2008) 711–716, <https://doi.org/10.1002/adma.200701530>.
- [14] S. Shan, S.H. Kang, J.R. Raney, P. Wang, L. Fang, F. Candido, J.A. Lewis, K. Bertoldi, Multistable architected materials for trapping elastic strain energy, *Adv. Mater.* 27 (2015) 4296–4301, <https://doi.org/10.1002/adma.201501708>.
- [15] S. Budday, P. Steinmann, E. Kuhl, The role of mechanics during brain development, *J. Mech. Phys. Solids* 72 (2014) 75–92, <https://doi.org/10.1016/j.jmps.2014.07.010>.
- [16] K.E. Garcia, C.D. Kroenke, P.V. Bayly, Mechanics of cortical folding: stress, growth and stability, *Philos. Trans. R. Soc. B Biol. Sci.* 373 (2018) 20170321, <https://doi.org/10.1098/rstb.2017.0321>.
- [17] Y. Du, Y. Su, L. Chaofeng, W. Chen, M. Destrade, Electro-mechanically guided growth and patterns, *J. Mech. Phys. Solids* (2020) 104073, <https://doi.org/10.1016/j.jmps.2020.104073>.
- [18] B. Li, Y.P. Cao, X.Q. Feng, H. Gao, Mechanics of morphological instabilities and surface wrinkling in soft materials: A review, *Soft Matter* 8 (2012) 5728–5745, <https://doi.org/10.1039/c2sm00011c>.
- [19] D. Ambrosi, M. Ben Amar, C.J. Cyron, A. DeSimone, A. Goriely, J.D. Humphrey, E. Kuhl, Growth and remodelling of living tissues: perspectives, challenges and opportunities, *J. R. Soc. Interface* 16 (2019) 20190233, <https://doi.org/10.1098/rsif.2019.0233>.
- [20] F. Xu, S. Zhao, Thermal wrinkling of liquid crystal polymer shell/core spheres, *Extrem. Mech. Lett.* 40 (2020) 100860, <https://doi.org/10.1016/j.eml.2020.100860>.
- [21] T. Lu, C. Ma, T. Wang, Mechanics of dielectric elastomer structures: A review, *Extrem. Mech. Lett.* 38 (2020) 100752, <https://doi.org/10.1016/j.eml.2020.100752>.
- [22] E. Psarra, L. Bodelot, K. Danas, Wrinkling to crinkling transitions and curvature localization in a magnetoelastic film bonded to a non-magnetic substrate, *J. Mech. Phys. Solids* 133 (2019) 103734, <https://doi.org/10.1016/j.jmps.2019.103734>.
- [23] E. Psarra, L. Bodelot, K. Danas, Two-field surface pattern control via marginally stable magnetorheological elastomers, *Soft Matter* 13 (2017) 6576–6584, <https://doi.org/10.1039/C7SM00996H>.
- [24] Leonhard Euler, Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici latissimo sensu accepti, Lausanne, Geneva, 1744.
- [25] Joseph-Louis Lagrange, Sur la figure des colonnes, *Misc. Taur.* 5 (1770) 123–166.
- [26] C. Coullais, J.T.B. Overvelde, L.A. Lubbers, K. Bertoldi, M. van Hecke, Discontinuous buckling of wide beams and metabeams, *Phys. Rev. Lett.* 115 (2015) 044301, <https://doi.org/10.1103/PhysRevLett.115.044301>.
- [27] Y. Chen, L. Jin, Snapping-back buckling of wide hyperelastic columns, *Extrem. Mech. Lett.* 34 (2020) 100600, <https://doi.org/10.1016/j.eml.2019.100600>.
- [28] Y. Chen, L. Jin, From continuous to snapping-back buckling: A post-buckling analysis for hyperelastic columns under axial compression, *Int. J. Non. Linear. Mech.* (2020) 103532, <https://doi.org/10.1016/j.ijnonlinmec.2020.103532>.
- [29] G. Oliveri, J.T.B. Overvelde, Inverse design of mechanical metamaterials that undergo buckling, *Adv. Funct. Mater.* 30 (2020) 1909033, <https://doi.org/10.1002/adfm.201909033>.
- [30] A. Lazarus, J.T. Miller, P.M. Reis, Continuation of equilibria and stability of slender elastic rods using an asymptotic numerical method, *J. Mech. Phys. Solids* 61 (2013) 1712–1736, <https://doi.org/10.1016/j.jmps.2013.04.002>.
- [31] J.T. Miller, T. Su, E.B. Dussan, V.J. Pabon, N. Wicks, K. Bertoldi, P.M. Reis, Buckling-induced lock-up of a slender rod injected into a horizontal cylinder, *Int. J. Solids Struct.* 72 (2015) 153–164, <https://doi.org/10.1016/j.ijsolstr.2015.07.025>.
- [32] J. Li, V. Slesarenko, P.I. Galich, S. Rudykh, Instabilities and pattern formations in 3D-printed deformable fiber composites, *Compos. Part B Eng.* 148 (2018) 114–122, <https://doi.org/10.1016/j.compositesb.2018.04.049>.
- [33] J.T. Miller, T. Su, J. Pabon, N. Wicks, K. Bertoldi, P.M. Reis, Buckling of a thin elastic rod inside a horizontal cylindrical constraint, *Extrem. Mech. Lett.* 3 (2015) 36–44, <https://doi.org/10.1016/j.eml.2015.03.002>.
- [34] J. Xiao, Y. Chen, X. Lu, B. Xu, X. Chen, J. Xu, Three dimensional buckling beam under cylindrical constraint, *Int. J. Mech. Sci.* 150 (2019) 348–355, <https://doi.org/10.1016/j.ijmecsci.2018.10.041>.
- [35] T. Su, J. Liu, D. Terwagne, P.M. Reis, K. Bertoldi, Buckling of an elastic rod embedded on an elastomeric matrix: planar vs. non-planar configurations, *Soft Matter* 10 (2014) 6294–6302, <https://doi.org/10.1039/c4sm00952e>.
- [36] Y. Zhao, J. Li, Y.P. Cao, X.Q. Feng, Buckling of an elastic fiber with finite length in a soft matrix, *Soft Matter* 12 (2016) 2086–2094, <https://doi.org/10.1039/c5sm02284c>.
- [37] Y. Chen, X. Liao, Y. Liu, X. Chen, Helical buckling of wires embedded in a soft matrix under axial compression, *Extrem. Mech. Lett.* 17 (2017) 71–76, <https://doi.org/10.1016/j.eml.2017.09.010>.
- [38] L.R. Herrmann, W.E. Mason, S.T.K. Chan, Response of reinforcing wires to compressive states of stress, *J. Compos. Mater.* 1 (1967) 212–226, <https://doi.org/10.1177/002199836700100301>.
- [39] B.W. Rosen, Mechanics of composite strengthening, in: *Fibre Composite Materials*, Am. Soc. Met., Ohio, 1965, pp. 37–75.
- [40] R. Parnes, A. Chiskis, Buckling of nano-fibre reinforced composites: a re-examination of elastic buckling, *J. Mech. Phys. Solids* 50 (2002) 855–879, [https://doi.org/10.1016/S0022-5096\(01\)00101-6](https://doi.org/10.1016/S0022-5096(01)00101-6).
- [41] R.W. Ogden, *Non-linear Elastic Deformations*, Dover Publications, New York, 1997.

- [42] N. Triantafyllidis, B.N. Maker, On the comparison between microscopic and macroscopic instability mechanisms in a class of fiber-reinforced composites, *J. Appl. Mech.* 52 (1985) 794–800, <https://doi.org/10.1115/1.3169148>.
- [43] G. Geymonat, S. Müller, N. Triantafyllidis, Homogenization of nonlinearly elastic materials, microscopic bifurcation and macroscopic loss of rank-one convexity, *Arch. Ration. Mech. Anal.* 122 (1993) 231–290, <https://doi.org/10.1007/BF00380256>.
- [44] J. Merodio, R.W. Ogden, Mechanical response of fiber-reinforced incompressible non-linearly elastic solids, *Int. J. Non. Linear. Mech.* 40 (2005) 213–227, <https://doi.org/10.1016/j.ijnonlinmec.2004.05.003>.
- [45] J. Merodio, R.W. Ogden, Material instabilities in fiber-reinforced nonlinearly elastic solids under plane deformation, *Arch. Mech.* 54 (2002) 525–552.
- [46] J. Merodio, R.W. Ogden, Instabilities and loss of ellipticity in fiber-reinforced compressible non-linearly elastic solids under plane deformation, *Int. J. Solids Struct.* 40 (2003) 4707–4727, <http://linkinghub.elsevier.com/retrieve/pii/S002722050000443>.
- [47] J. Merodio, R.W. Ogden, Remarks on instabilities and ellipticity for a fiber-reinforced compressible nonlinearly elastic solid under plane deformation, *Quart. Appl. Math.* 63 (2005) 325–333.
- [48] M. El Hamdaoui, J. Merodio, R.W. Ogden, Two-phase piecewise homogeneous plane deformations of a fibre-reinforced neo-Hookean material with application to fibre kinking and splitting, *J. Mech. Phys. Solids* 143 (2020) 104091, <https://doi.org/10.1016/j.jmps.2020.104091>.
- [49] J. Rodríguez, J. Merodio, Helical buckling and postbuckling of pre-stressed cylindrical tubes under finite torsion, *Finite Elem. Anal. Des.* 112 (2016) 1–10, <https://doi.org/10.1016/j.finel.2015.12.003>.
- [50] H. Demirkoparan, J. Merodio, Swelling and axial propagation of bulging with application to aneurysm propagation in arteries, *Math. Mech. Solids* 25 (2020) 1459–1471, <https://doi.org/10.1177/1081286517717127>.
- [51] M. Agoras, O. Lopez-Pamies, P. Ponte Castañeda, Onset of macroscopic instabilities in fiber-reinforced elastomers at finite strain, *J. Mech. Phys. Solids* 57 (2009) 1828–1850, <https://doi.org/10.1016/j.jmps.2009.08.001>.
- [52] S. Rudykh, G. DeBotton, Instabilities of hyperelastic fiber composites: micromechanical versus numerical analyses, *J. Elast.* 106 (2012) 123–147, <https://doi.org/10.1007/s10659-011-9313-x>.
- [53] J. Li, V. Slesarenko, S. Rudykh, Microscopic instabilities and elastic wave propagation in finitely deformed laminates with compressible hyperelastic phases, *Eur. J. Mech. – A/Solids* 73 (2019) 126–136, <https://doi.org/10.1016/j.euromechsol.2018.07.004>.
- [54] V. Slesarenko, S. Rudykh, Microscopic and macroscopic instabilities in hyperelastic fiber composites, *J. Mech. Phys. Solids* 99 (2017) 471–482, <https://doi.org/10.1016/j.jmps.2016.11.002>.
- [55] P.I. Galich, V. Slesarenko, J. Li, S. Rudykh, Elastic instabilities and shear waves in hyperelastic composites with various periodic fiber arrangements, *Int. J. Eng. Sci.* 130 (2018) 51–61, <https://doi.org/10.1016/j.ijengsci.2018.05.003>.
- [56] N. Arora, J. Li, V. Slesarenko, S. Rudykh, Microscopic and long-wave instabilities in 3D fiber composites with non-Gaussian hyperelastic phases, *Int. J. Eng. Sci.* 157 (2020) 103408, <https://doi.org/10.1016/j.ijengsci.2020.103408>.
- [57] F. Greco, L. Leonetti, C.M. Medaglia, R. Penna, A. Pranno, Nonlinear compressive failure analysis of biaxially loaded fiber reinforced materials, *Compos. Part B Eng.* 147 (2018) 240–251, <https://doi.org/10.1016/j.compositesb.2018.04.006>.
- [58] F. Greco, P. Lonetti, R. Luciano, P. Nevone Blasi, A. Pranno, Nonlinear effects in fracture induced failure of compressively loaded fiber reinforced composites, *Compos. Struct.* 189 (2018) 688–699, <https://doi.org/10.1016/j.compstruct.2018.01.014>.
- [59] Y. Li, N. Kaynia, S. Rudykh, M.C. Boyce, Wrinkling of interfacial layers in stratified composites, *Adv. Eng. Mater.* 15 (2013) 921–926, <https://doi.org/10.1002/adem.201200387>.
- [60] V. Slesarenko, S. Rudykh, Harnessing viscoelasticity and instabilities for tuning wavy patterns in soft layered composites, *Soft Matter* 12 (2016) 3677–3682, <https://doi.org/10.1039/C5SM02949J>.
- [61] N. Arora, A. Batan, J. Li, V. Slesarenko, S. Rudykh, On the influence of inhomogeneous interphase layers on instabilities in hyperelastic composites, *Materials* 12 (2019) 763, <https://doi.org/10.3390/ma12050763>.
- [62] J. Li, T.D. Pallicity, V. Slesarenko, A. Goshkoderia, S. Rudykh, Domain formations and pattern transitions via instabilities in soft heterogeneous materials, *Adv. Mater.* 31 (2019) 1807309, <https://doi.org/10.1002/adma.201807309>.
- [63] J.C. Michel, O. Lopez-Pamies, P. Ponte Castañeda, N. Triantafyllidis, Microscopic and macroscopic instabilities in finitely strained fiber-reinforced elastomers, *J. Mech. Phys. Solids* 58 (2010) 1776–1803, <https://doi.org/10.1016/j.jmps.2010.08.006>.
- [64] R. Avazmohammadi, P. Ponte Castañeda, Macroscopic constitutive relations for elastomers reinforced with short aligned fibers: Instabilities and post-bifurcation response, *J. Mech. Phys. Solids* 97 (2016) 37–67, <https://doi.org/10.1016/j.jmps.2015.07.007>.
- [65] N. Triantafyllidis, M.D. Nestorović, M.W. Schraad, Failure surfaces for finitely strained two-phase periodic solids under general in-plane loading, *J. Appl. Mech.* 73 (2006) 505–515, <https://doi.org/10.1115/1.2126695>.
- [66] X. Chen, J.W. Hutchinson, Herringbone buckling patterns of compressed thin films on compliant substrates, *J. Appl. Mech.* 71 (2004) 597, <https://doi.org/10.1115/1.1756141>.
- [67] J. Song, H. Jiang, W.M. Choi, D.Y. Khang, Y. Huang, J.A. Rogers, An analytical study of two-dimensional buckling of thin films on compliant substrates, *J. Appl. Phys.* 103 (2008) 014303, <https://doi.org/10.1063/1.2828050>.
- [68] S. Cai, D. Breid, A.J. Crosby, Z. Suo, J.W. Hutchinson, Periodic patterns and energy states of buckled films on compliant substrates, *J. Mech. Phys. Solids* 59 (2011) 1094–1114, <https://doi.org/10.1016/j.jmps.2011.02.001>.
- [69] L. Tran, M.O. Lavrentovich, G. Durey, A. Darmon, M.F. Haase, N. Li, D. Lee, K. J. Stebe, R.D. Kamien, T. Lopez-Leon, Change in stripes for cholesteric shells via anchoring in moderation, *Phys. Rev. X* 7 (2017) 041029, <https://doi.org/10.1103/PhysRevX.7.041029>.
- [70] G.C. Verwey, M. Warner, E.M. Terentjev, Elastic instability and stripe domains in liquid crystalline elastomers, *J. Phys. II* (6) (1996) 1273–1290, <https://doi.org/10.1051/jp2:1996130>.
- [71] H. Finkelmann, I. Kundler, E.M. Terentjev, M. Warner, Critical stripe-domain instability of nematic elastomers, *J. Phys. II* 7 (1997) 1059–1069, <https://doi.org/10.1051/jp2:1997171>.
- [72] S. Rudykh, A. Lewinstein, G. Uner, G. DeBotton, Analysis of microstructural induced enhancement of electromechanical coupling in soft dielectrics, *Appl. Phys. Lett.* 102 (2013) 2012–2015, <https://doi.org/10.1063/1.4801775>.
- [73] S. Rudykh, K. Bhattacharya, G. DeBotton, Snap-through actuation of thick-wall electroactive balloons, *Int. J. Non. Linear. Mech.* 47 (2012) 206–209, <https://doi.org/10.1016/j.ijnonlinmec.2011.05.006>.
- [74] A. Goshkoderia, S. Rudykh, Stability of magnetoactive composites with periodic microstructures undergoing finite strains in the presence of a magnetic field, *Compos. Part B Eng.* 128 (2017) 19–29, <https://doi.org/10.1016/j.compositesb.2017.06.014>.
- [75] A. Goshkoderia, S. Rudykh, Electromechanical macroscopic instabilities in soft dielectric elastomer composites with periodic microstructures, *Eur. J. Mech. A/Solids* 65 (2017) 243–256, <https://doi.org/10.1016/j.euromechsol.2017.04.008>.
- [76] A. Goshkoderia, V. Chen, J. Li, A. Juhl, P. Buskohl, S. Rudykh, Instability-induced pattern formations in soft magnetoactive composites, *Phys. Rev. Lett.* 124 (2020) 158002, <https://doi.org/10.1103/PhysRevLett.124.158002>.
- [77] A. Goshkoderia, N. Arora, V. Slesarenko, J. Li, V. Chen, A. Juhl, P. Buskohl, S. Rudykh, Tunable permittivity in dielectric elastomer composites under finite strains: Periodicity, randomness, and instabilities, *Int. J. Mech. Sci.* 186 (2020) 105880, <https://doi.org/10.1016/j.ijmecsci.2020.105880>.
- [78] E. Polukhov, D. Vallicotti, M.A. Keip, Computational stability analysis of periodic electroactive polymer composites across scales, *Comput. Methods Appl. Mech. Eng.* 337 (2018) 165–197, <https://doi.org/10.1016/j.cma.2018.01.020>.
- [79] M. Jandron, D.L. Henann, Electromechanical instabilities in periodic dielectric elastomer composites, *Int. J. Solids Struct.* 191–192 (2020) 220–242, <https://doi.org/10.1016/j.jisolsstr.2019.12.008>.
- [80] Y. Su, B. Wu, W. Chen, M. Destade, Pattern evolution in bending dielectric-elastomeric bilayers, *J. Mech. Phys. Solids* 136 (2020) 103670, <https://doi.org/10.1016/j.jmps.2019.07.013>.
- [81] O. Lopez-Pamies, P.P. Castañeda, Second-order estimates for the macroscopic response and loss of ellipticity in porous rubbers at large deformations, *J. Elast.* 76 (2004) 247–287, <https://doi.org/10.1007/s10659-005-1405-z>.
- [82] O. Lopez-Pamies, P. Ponte Castañeda, Homogenization-based constitutive models for porous elastomers and implications for macroscopic instabilities: II—Results, *J. Mech. Phys. Solids* 55 (2007) 1702–1728, <https://doi.org/10.1016/j.jmps.2007.01.008>.
- [83] J.C. Michel, O. Lopez-Pamies, P. Ponte Castañeda, N. Triantafyllidis, Microscopic and macroscopic instabilities in finitely strained porous elastomers, *J. Mech. Phys. Solids* 55 (2007) 900–938, <https://doi.org/10.1016/j.jmps.2010.08.006>.
- [84] R. Abeyaratne, N. Triantafyllidis, An investigation of localization in a porous elastic material using homogenization theory, *J. Appl. Mech.* 51 (1984) 481–485, <https://doi.org/10.1115/1.3167661>.
- [85] K. Bertoldi, M.C. Boyce, Wave propagation and instabilities in monolithic and periodically structured elastomeric materials undergoing large deformations, *Phys. Rev. B* 78 (2008) 184107, <https://doi.org/10.1103/PhysRevB.78.184107>.
- [86] K. Bertoldi, M.C. Boyce, S. Deschanel, S.M. Prange, T. Mullin, Mechanics of deformation-triggered pattern transformations and superelastic behavior in periodic elastomeric structures, *J. Mech. Phys. Solids* 56 (2008) 2642–2668, <https://doi.org/10.1016/j.jmps.2008.03.006>.
- [87] T. Mullin, S. Deschanel, K. Bertoldi, M.C. Boyce, Pattern transformation triggered by deformation, *Phys. Rev. Lett.* 99 (2007) 084301, <https://doi.org/10.1103/PhysRevLett.99.084301>.
- [88] S. Singamaneni, K. Bertoldi, S. Chang, J.H. Jang, S.L. Young, E.L. Thomas, M. C. Boyce, V.V. Tsukruk, Bifurcated mechanical behavior of deformed periodic porous solids, *Adv. Funct. Mater.* 19 (2009) 1426–1436, <https://doi.org/10.1002/adfm.200801675>.
- [89] K. Bertoldi, P.M. Reis, S. Willshaw, T. Mullin, Negative Poisson's ratio behavior induced by an elastic instability, *Adv. Mater.* 22 (2010) 361–366, <https://doi.org/10.1002/adma.200901956>.
- [90] S. Babaee, J. Shim, J.C. Weaver, E.R. Chen, N. Patel, K. Bertoldi, 3D soft metamaterials with negative Poisson's ratio, *Adv. Mater.* 25 (2013) 5044–5049, <https://doi.org/10.1002/adma.201301986>.
- [91] J.T.B. Overvelde, S. Shan, K. Bertoldi, Compaction through buckling in 2D periodic, soft and porous structures: Effect of pore shape, *Adv. Mater.* 24 (2012) 2337–2342, <https://doi.org/10.1002/adma.201104395>.
- [92] J.T.B. Overvelde, K. Bertoldi, Relating pore shape to the non-linear response of periodic elastomeric structures, *J. Mech. Phys. Solids* 64 (2014) 351–366, <https://doi.org/10.1016/j.jmps.2013.11.014>.
- [93] A. Clausen, F. Wang, J.S. Jensen, O. Sigmund, J.A. Lewis, Topology optimized architectures with programmable Poisson's ratio over large deformations, *Adv. Mater.* 27 (2015) 5523–5527, <https://doi.org/10.1002/adma.201502485>.

- [94] J. Shim, S. Shan, A. Kosmrlj, S.H. Kang, E.R. Chen, J.C. Weaver, K. Bertoldi, Harnessing instabilities for design of soft reconfigurable auxetic/chiral materials, *Soft Matter* 9 (2013) 8198, <https://doi.org/10.1039/c3sm51148k>.
- [95] J. Shim, C. Perdigou, E.R. Chen, K. Bertoldi, P.M. Reis, Buckling-induced encapsulation of structured elastic shells under pressure, *Proc. Natl. Acad. Sci.* 109 (2012) 5978–5983, <https://doi.org/10.1073/pnas.1115674109>.
- [96] J. Li, S. Rudykh, Tunable microstructure transformations and auxetic behavior in 3D-printed multiphase composites: The role of inclusion distribution, *Compos. Part B Eng.* 172 (2019) 352–362, <https://doi.org/10.1016/j.compositesb.2019.05.012>.
- [97] J. Li, V. Slesarenko, S. Rudykh, Auxetic multiphase soft composite material design through instabilities with application for acoustic metamaterials, *Soft Matter* 14 (2018) 6171–6180, <https://doi.org/10.1039/C8SM00874D>.
- [98] J. Li, Y. Wang, W. Chen, Y.S. Wang, R. Bao, Harnessing inclusions to tune post-buckling deformation and bandgaps of soft porous periodic structures, *J. Sound Vib.* 459 (2019) 114848, <https://doi.org/10.1016/j.jsv.2019.114848>.
- [99] B. Florijn, C. Coulais, M. van Hecke, Programmable mechanical metamaterials, *Phys. Rev. Lett.* 113 (2014) 175503, <https://doi.org/10.1103/PhysRevLett.113.175503>.
- [100] B. Florijn, C. Coulais, M. van Hecke, Programmable mechanical metamaterials: the role of geometry, *Soft Matter* 12 (2016) 8736–8743, <https://doi.org/10.1039/C6SM01271J>.
- [101] J.T.B. Overvelde, D.M.J. Dykstra, R. de Rooij, J. Weaver, K. Bertoldi, Tensile instability in a thick elastic body, *Phys. Rev. Lett.* 117 (2016) 094301, <https://doi.org/10.1103/PhysRevLett.117.094301>.
- [102] A. Fargette, S. Neukirch, A. Antkowiak, Elastocapillary snapping: Capillarity induces snap-through instabilities in small elastic beams, *Phys. Rev. Lett.* 112 (2014) 137802, <https://doi.org/10.1103/PhysRevLett.112.137802>.
- [103] A. Pandey, D.E. Moulton, D. Vella, D.P. Holmes, Dynamics of snapping beams and jumping poppers, *Europhys. Lett.* 105 (2014) 24001, <https://doi.org/10.1209/0295-5075/105/24001>.
- [104] D. Restrepo, N.D. Mankame, P.D. Zavattieri, Phase transforming cellular materials, *Extrem. Mech. Lett.* 4 (2015) 52–60, <https://doi.org/10.1016/j.eml.2015.08.001>.
- [105] A. Rafsanjani, A. Akbarzadeh, D. Pasini, Snapping mechanical metamaterials under tension, *Adv. Mater.* 27 (2015) 5931–5935, <https://doi.org/10.1002/adma.201502809>.
- [106] M. Ryykin, V. Slesarenko, A. Cherkaev, S. Rudykh, Fault-tolerant elastic-plastic lattice material, *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.* 378 (2020) 20190107, <https://doi.org/10.1098/rsta.2019.0107>.
- [107] M.C. Fernandes, J. Aizenberg, J.C. Weaver, K. Bertoldi, Mechanically robust lattices inspired by deep-sea glass sponges, *Nat. Mater.* (2020), <https://doi.org/10.1038/s41563-020-0798-1>.
- [108] K. Che, C. Yuan, J. Wu, H.J. Qi, J. Meaud, Three-dimensional-printed multistable mechanical metamaterials with a deterministic deformation sequence, *J. Appl. Mech.* 84 (2016) 011004, <https://doi.org/10.1115/1.4034706>.
- [109] B. Haghighpanah, J. Papadopoulos, D. Mousanezhad, H. Nayeib-Hashemi, A. Vaziri, Buckling of regular, chiral and hierarchical honeycombs under a general macroscopic stress state, *Proc. R. Soc. A Math. Phys. Eng. Sci.* 470 (2014) 20130856, <https://doi.org/10.1098/rspa.2013.0856>.
- [110] T. Frenzel, C. Findeisen, M. Kadic, P. Gumbsch, M. Wegener, Tailored buckling microlattices as reusable light-weight shock absorbers, *Adv. Mater.* 28 (2016) 5865–5870, <https://doi.org/10.1002/adma.201600610>.
- [111] C. Coulais, E. Teomy, K. De Reus, Y. Shokef, M. Van Hecke, Combinatorial design of textured mechanical metamaterials, *Nature* 535 (2016) 529–532, <https://doi.org/10.1038/nature18960>.
- [112] D.J. Rayneau-Kirkhope, M.A. Dias, Recipes for selecting failure modes in 2-d lattices, *Extrem. Mech. Lett.* 9 (2016) 11–20, <https://doi.org/10.1016/j.eml.2016.04.004>.
- [113] S.H. Kang, S. Shan, A. Kosmrlj, W.L. Noorduin, S. Shian, J.C. Weaver, D.R. Clarke, K. Bertoldi, Complex ordered patterns in mechanical instability induced geometrically frustrated triangular cellular structures, *Phys. Rev. Lett.* 112 (2014) 098701, <https://doi.org/10.1103/PhysRevLett.112.098701>.
- [114] C. Yuan, X. Mu, C.K. Dunn, J. Haidar, T. Wang, H.J. Qi, Thermomechanically triggered two-stage pattern switching of 2D lattices for adaptive structures, *Adv. Funct. Mater.* 28 (2018) 1705727, <https://doi.org/10.1002/adfm.201705727>.
- [115] X. Xia, A. Afshar, H. Yang, C.M. Portela, D.M. Kochmann, C.V. Di Leo, J.R. Greer, Electrochemically reconfigurable architected materials, *Nature* 573 (2019) 205–213, <https://doi.org/10.1038/s41586-019-1538-z>.
- [116] S.H. Kang, S. Shan, W.L. Noorduin, M. Khan, J. Aizenberg, K. Bertoldi, Buckling-induced reversible symmetry breaking and amplification of chirality using supported cellular structures, *Adv. Mater.* 25 (2013) 3380–3385, <https://doi.org/10.1002/adma.201300617>.
- [117] J. Liu, T. Gu, S. Shan, S.H. Kang, J.C. Weaver, K. Bertoldi, Harnessing buckling to design architected materials that exhibit effective negative swelling, *Adv. Mater.* 28 (2016) 6619–6624, <https://doi.org/10.1002/adma.201600812>.
- [118] C. Findeisen, J. Hohe, M. Kadic, P. Gumbsch, Characteristics of mechanical metamaterials based on buckling elements, *J. Mech. Phys. Solids* 102 (2017) 151–164, <https://doi.org/10.1016/j.jmps.2017.02.011>.
- [119] D.P. Holmes, Elasticity and stability of shape-shifting structures, *Curr. Opin. Colloid Interface Sci.* 40 (2019) 118–137, <https://doi.org/10.1016/j.cocis.2019.02.008>.
- [120] M.A. Biot, Surface instability of rubber in compression, *Appl. Sci. Res.* 12 (1963) 168–182, <https://doi.org/10.1007/BF03184638>.
- [121] Y. Cao, J.W. Hutchinson, From wrinkles to creases in elastomers: The instability and imperfection-sensitivity of wrinkling, *Proc. R. Soc. A Math. Phys. Eng. Sci.* 468 (2012) 94–115, <https://doi.org/10.1098/rspa.2011.0384>.
- [122] Z.Y. Huang, W. Hong, Z. Suo, Nonlinear analyses of wrinkles in a film bonded to a compliant substrate, *J. Mech. Phys. Solids* 53 (2005) 2101–2118, <https://doi.org/10.1016/j.jmps.2005.03.007>.
- [123] C.M. Stafford, B.D. Vogt, C. Harrison, D. Julthongpiput, R. Huang, Elastic moduli of ultrathin amorphous polymer films, *Macromolecules* 39 (2006) 5095–5099, <https://doi.org/10.1021/ma060790i>.
- [124] H. Jiang, D.-Y. Khang, J. Song, Y. Sun, Y. Huang, J.A. Rogers, Finite deformation mechanics in buckled thin films on compliant supports, *Proc. Natl. Acad. Sci.* 104 (2007) 15607–15612, <https://doi.org/10.1073/pnas.0702927104>.
- [125] D.-Y. Khang, H. Jiang, Y. Huang, J.A. Rogers, A stretchable form of single-crystal, *Science* 311 (2006) 208–212, <https://doi.org/10.1126/science.1121401>.
- [126] J. Song, H. Jiang, Z.J. Liu, D.Y. Khang, Y. Huang, J.A. Rogers, C. Lu, C.G. Koh, Buckling of a stiff thin film on a compliant substrate in large deformation, *Int. J. Solids Struct.* 45 (2008) 3107–3121, <https://doi.org/10.1016/j.ijsolstr.2008.01.023>.
- [127] H. Cheng, J. Song, A simply analytic study of buckled thin films on compliant substrates, *J. Appl. Mech.* 81 (2014) 024501, <https://doi.org/10.1115/1.4025306>.
- [128] Y.-C. Chen, A.J. Crosby, High aspect ratio wrinkles via substrate prestretch, *Adv. Mater.* 26 (2014) 5626–5631, <https://doi.org/10.1002/adma.201401444>.
- [129] J.Y. Sun, S. Xia, M.W. Moon, K.H. Oh, K.S. Kim, Folding wrinkles of a thin stiff layer on a soft substrate, *Proc. R. Soc. A Math. Phys. Eng. Sci.* 468 (2012) 932–953, <https://doi.org/10.1098/rspa.2011.0567>.
- [130] L. Pocivavsek, R. Dellsy, A. Kern, S. Johnson, B. Lin, K.Y.C. Lee, E. Cerda, Stress and fold localization in thin elastic membranes, *Science* 320 (2008) 912–916, <https://doi.org/10.1126/science.1154069>.
- [131] F. Brau, P. Damman, H. Diamant, T.A. Witten, Wrinkle to fold transition: Influence of the substrate response, *Soft Matter* 9 (2013) 8177–8186, <https://doi.org/10.1039/c3sm50655j>.
- [132] Y. Cao, J.W. Hutchinson, Wrinkling phenomena in neo-hookean film/substrate bilayers, *J. Appl. Mech.* 79 (2012) 031019, <https://doi.org/10.1115/1.4005960>.
- [133] L. Jin, A. Takei, J.W. Hutchinson, Mechanics of wrinkle/ridge transitions in thin film/substrate systems, *J. Mech. Phys. Solids* 81 (2015) 22–40, <https://doi.org/10.1016/j.jmps.2015.04.016>.
- [134] F. Brau, H. Vandeparre, A. Sabbah, C. Poullard, A. Boudaoud, P. Damman, Multiple-length-scale elastic instability mimics parametric resonance of nonlinear oscillators, *Nat. Phys.* 7 (2011) 56–60, <https://doi.org/10.1038/nphys1806>.
- [135] B. Li, Y.P. Cao, X.Q. Feng, H. Gao, Surface wrinkling of mucosa induced by volumetric growth: Theory, simulation and experiment, *J. Mech. Phys. Solids* 59 (2011) 758–774, <https://doi.org/10.1016/j.jmps.2011.01.010>.
- [136] A. Auguste, J. Yang, L. Jin, D. Chen, Z. Suo, R.C. Hayward, Formation of high aspect ratio wrinkles and ridges on elastic bilayers with small thickness contrast, *Soft Matter* 14 (2018) 8545–8551, <https://doi.org/10.1039/c8sm01345d>.
- [137] K. Efimenko, M. Rackaitis, E. Manias, A. Vaziri, L. Mahadevan, J. Genzer, Nested self-similar wrinkling patterns in skins, *Nat. Mater.* 4 (2005) 293–297, <https://doi.org/10.1038/nmat1342>.
- [138] W.T.S. Huck, Hierarchical wrinkling, *Nat. Mater.* 4 (2005) 271–272, <https://doi.org/10.1038/nmat1356>.
- [139] J.Y. Chung, J.H. Lee, K.L. Beers, C.M. Stafford, Stiffness, strength, and ductility of nanoscale thin films and membranes: A combined wrinkling-cracking methodology, *Nano Lett.* 11 (2011) 3361–3365, <https://doi.org/10.1021/nl201764b>.
- [140] D. Vella, J. Bico, A. Boudaoud, B. Roman, P.M. Reis, The macroscopic delamination of thin films from elastic substrates, *Proc. Natl. Acad. Sci. U. S. A.* 106 (2009) 10901–10906, <https://doi.org/10.1073/pnas.0902160106>.
- [141] Y. Ebata, A.B. Croll, A.J. Crosby, Wrinkling and strain localizations in polymer thin films, *Soft Matter* 8 (2012) 9086–9091, <https://doi.org/10.1039/c2sm25859e>.
- [142] H. Mei, C.M. Landis, R. Huang, Concomitant wrinkling and buckle-delamination of elastic thin films on compliant substrates, *Mech. Mater.* 43 (2011) 627–642, <https://doi.org/10.1016/j.mechmat.2011.08.003>.
- [143] A. Auguste, L. Jin, Z. Suo, R.C. Hayward, Post-wrinkle bifurcations in elastic bilayers with modest contrast in modulus, *Extrem. Mech. Lett.* 11 (2017) 30–36, <https://doi.org/10.1016/j.eml.2016.11.013>.
- [144] L. Jin, A. Auguste, R.C. Hayward, Z. Suo, Bifurcation diagrams for the formation of wrinkles or creases in Soft Bilayers, *J. Appl. Mech. Trans. ASME* 82 (2015) 061008, <https://doi.org/10.1115/1.4030384>.
- [145] Q. Wang, X. Zhao, A three-dimensional phase diagram of growth-induced surface instabilities, *Sci. Rep.* 5 (2015) 8887, <https://doi.org/10.1038/srep08887>.
- [146] Q. Wang, X. Zhao, Phase diagrams of instabilities in compressed film-substrate systems, *J. Appl. Mech. Trans. ASME* 81 (2014), 051004, <https://doi.org/10.1115/1.4025828>.
- [147] Q. Wang, X. Zhao, Beyond wrinkles: Multimodal surface instabilities for multifunctional patterning, *MRS Bull.* 41 (2016) 115–122, <https://doi.org/10.1557/mrs.2015.338>.
- [148] J. Yin, J.L. Yagüe, D. Eggenspieler, K.K. Gleason, M.C. Boyce, Deterministic order in surface micro-topologies through sequential wrinkling, *Adv. Mater.* 24 (2012) 5441–5446, <https://doi.org/10.1002/adma.201201937>.
- [149] W.M. Choi, J. Song, D.Y. Khang, H. Jiang, Y.Y. Huang, J.A. Rogers, Biaxially stretchable “wavy” silicon nanomembranes, *Nano Lett.* 7 (2007) 1655–1663, <https://doi.org/10.1021/nl0706244>.

- [150] S.F. Yin, B. Li, Y.P. Cao, X.Q. Feng, Surface wrinkling of anisotropic films bonded on a compliant substrate, *Int. J. Solids Struct.* 141–142 (2018) 219–231, <https://doi.org/10.1016/j.jssolstr.2018.02.024>.
- [151] H.J. Bae, S. Bae, C. Park, S. Han, J. Kim, L.N. Kim, K. Kim, S.-H. Song, W. Park, S. Kwon, Biomimetic microfingerprints for anti-counterfeiting strategies, *Adv. Mater.* 27 (2015) 2083–2089, <https://doi.org/10.1002/adma.201405483>.
- [152] J. Yin, M.C. Boyce, Unique wrinkles as identity tags, *Nature* 520 (2015) 164–165, <https://doi.org/10.1038/nature14380>.
- [153] P. Kim, M. Abkarian, H.A. Stone, Hierarchical folding of elastic membranes under biaxial compressive stress, *Nat. Mater.* 10 (2011) 952–957, <https://doi.org/10.1038/nmat3144>.
- [154] P.C. Lin, S. Yang, Spontaneous formation of one-dimensional ripples in transit to highly ordered two-dimensional herringbone structures through sequential and unequal biaxial mechanical stretching, *Appl. Phys. Lett.* 90 (2007) 241903, <https://doi.org/10.1063/1.2743939>.
- [155] B. Audoly, A. Boudaoud, Buckling of a stiff film bound to a compliant substrate—Part III: Herringbone solutions at large buckling parameter, *J. Mech. Phys. Solids* 56 (2008) 2444–2458, <https://doi.org/10.1016/j.jmps.2008.03.001>.
- [156] B. Audoly, A. Boudaoud, Buckling of a stiff film bound to a compliant substrate—Part II: A global scenario for the formation of herringbone pattern, *J. Mech. Phys. Solids* 56 (2008) 2422–2443, <https://doi.org/10.1016/j.jmps.2008.03.001>.
- [157] B. Audoly, A. Boudaoud, Buckling of a stiff film bound to a compliant substrate—Part I: Formulation, linear stability of cylindrical patterns, secondary bifurcations, *J. Mech. Phys. Solids* 56 (2008) 2401–2421, <https://doi.org/10.1016/j.jmps.2008.03.001>.
- [158] X. Chen, J.W. Hutchinson, A family of herringbone patterns in thin films, *Scr. Mater.* 50 (2004) 797–801, <https://doi.org/10.1016/j.scriptamat.2003.11.035>.
- [159] N. Bowden, S. Brittain, A.G. Evans, J.W. Hutchinson, G.M. Whitesides, Spontaneous formation of ordered structures in thin films of metals supported on an elastomeric polymer, *Nature* 393 (1998) 146–149, <https://doi.org/10.1038/30193>.
- [160] Y. Tan, B. Hu, J. Song, Z. Chu, W. Wu, Bioinspired multiscale wrinkling patterns on curved substrates: An overview, *Nano-Micro Lett.* 12 (2020), <https://doi.org/10.1007/s40820-020-00436-y>.
- [161] D.H. Kim, J. Song, M.C. Won, H.S. Kim, R.H. Kim, Z. Liu, Y.Y. Huang, K.C. Hwang, Y.W. Zhang, J.A. Rogers, Materials and noncoplanar mesh designs for integrated circuits with linear elastic responses to extreme mechanical deformations, *Proc. Natl. Acad. Sci. U. S. A.* 105 (2008) 18675–18680, <https://doi.org/10.1073/pnas.0807476105>.
- [162] J. Song, Y. Huang, J. Xiao, S. Wang, K.C. Hwang, H.C. Ko, D.-H. Kim, M. P. Stoykovich, J.A. Rogers, Mechanics of noncoplanar mesh design for stretchable electronic circuits, *J. Appl. Phys.* 105 (2009) 123516, <https://doi.org/10.1063/1.3148245>.
- [163] S. Xu, Z. Yan, K.I. Jang, W. Huang, H. Fu, J. Kim, Z. Wei, M. Flavin, J. McCracken, R. Wang, A. Badea, Y. Liu, D. Xiao, G. Zhou, J. Lee, H.U. Chung, H. Cheng, W. Ren, A. Banks, X. Li, U. Paik, R.G. Nuzzo, Y. Huang, Y. Zhang, J.A. Rogers, Assembly of micro/nanomaterials into complex, three-dimensional architectures by compressive buckling, *Science* 347 (2015) 154–159, <https://doi.org/10.1126/science.1260960>.
- [164] H. Fu, K. Nan, W. Bai, W. Huang, K. Bai, L. Lu, C. Zhou, Y. Liu, F. Liu, J. Wang, M. Han, Z. Yan, H. Luan, Y. Zhang, Y. Zhang, J. Zhao, X. Cheng, M. Li, J.W. Lee, Y. Liu, D. Fang, X. Li, Y. Huang, Y. Zhang, J.A. Rogers, Morphable 3D mesostructures and microelectronic devices by multistable buckling mechanics, *Nat. Mater.* 17 (2018) 268–276, <https://doi.org/10.1038/s41563-017-0011-3>.
- [165] H. Zhao, K. Li, M. Han, F. Zhu, A. Vázquez-Guardado, P. Guo, Z. Xie, Y. Park, L. Chen, X. Wang, H. Luan, Y. Yang, H. Wang, C. Liang, Y. Xue, R.D. Schaller, D. Chanda, Y. Huang, Y. Zhang, J.A. Rogers, Buckling and twisting of advanced materials into morphable 3D mesostructures, *Proc. Natl. Acad. Sci. U. S. A.* 116 (2019) 13239–13248, <https://doi.org/10.1073/pnas.1901193116>.
- [166] Z. Fan, Y. Yang, F. Zhang, Z. Xu, H. Zhao, T. Wang, H. Song, Y. Huang, J. A. Rogers, Y. Zhang, Inverse design strategies for 3D surfaces formed by mechanically guided assembly, *Adv. Mater.* 32 (2020) 1908424, <https://doi.org/10.1002/adma.201908424>.
- [167] J. Song, Mechanics of stretchable electronics, *Curr. Opin. Solid State Mater. Sci.* 19 (2015) 160–170, <https://doi.org/10.1016/j.cossms.2015.01.004>.
- [168] Y. Zhang, F. Zhang, Z. Yan, Q. Ma, X. Li, Y. Huang, J.A. Rogers, Printing, folding and assembly methods for forming 3D mesostructures in advanced materials, *Nat. Rev. Mater.* 2 (2017) 17019, <https://doi.org/10.1038/natrevmats.2017.19>.
- [169] H. Fu, K. Bai, Y. Huang, Y. Zhang, Recent progress of morphable 3D mesostructures in advanced materials, *J. Semicond.* 41 (2020) 041604, <https://doi.org/10.1088/1674-4926/41/4/041604>.
- [170] A.N. Gent, I.S. Cho, Surface instabilities in compressed or bent rubber blocks, *Rubber Chem. Technol.* 72 (1999) 253–262, <https://doi.org/10.5254/1.3538798>.
- [171] W. Hong, X. Zhao, Z. Suo, Formation of creases on the surfaces of elastomers and gels, *Appl. Phys. Lett.* 95 (2009) 111901, <https://doi.org/10.1063/1.3211917>.
- [172] S. Cai, D. Chen, Z. Suo, R.C. Hayward, Creasing instability of elastomer films, *Soft Matter* 8 (2012) 1301–1304, <https://doi.org/10.1039/c2sm06844c>.
- [173] K. Saha, J. Kim, E. Irwin, J. Yoon, F. Momin, V. Trujillo, D.V. Schaffer, K.E. Healy, R.C. Hayward, Surface creasing instability of soft polyacrylamide cell culture substrates, *Biophys. J.* 99 (2010) 94–96, <https://doi.org/10.1016/j.bpj.2010.09.045>.
- [174] P. Ciarletta, Matched asymptotic solution for crease nucleation in soft solids, *Nat. Commun.* 9 (2018) 496, <https://doi.org/10.1038/s41467-018-02979-6>.
- [175] D. Chen, L. Jin, Z. Suo, R.C. Hayward, Controlled formation and disappearance of creases, *Mater. Horiz.* 1 (2014) 207–213, <https://doi.org/10.1039/C3MH00107E>.
- [176] V. Trujillo, J. Kim, R.C. Hayward, Creasing instability of surface-attached hydrogels, *Soft Matter* 4 (2008) 564–569, <https://doi.org/10.1039/b713263h>.
- [177] J. Yoon, J. Kim, R.C. Hayward, Nucleation, growth, and hysteresis of surface creases on swelled polymer gels, *Soft Matter* 6 (2010) 5807–5816, <https://doi.org/10.1039/c0sm00372g>.
- [178] J. Kim, J. Yoon, R.C. Hayward, Dynamic display of biomolecular patterns throughanelastic creasing instability of stimuli-responsive hydrogels, *Nat. Mater.* 9 (2010) 159–164, <https://doi.org/10.1038/nmat2606>.
- [179] M.P. Milner, L. Jin, S.B. Hutchens, Creasing in evaporation-driven cavity collapse, *Soft Matter* 13 (2017) 6894–6904, <https://doi.org/10.1039/c7sm01258f>.
- [180] J. Dervaux, Y. Couder, M.-A. Guedeau-Boudeville, M. Ben Amar, Shape transition in artificial tumors: From smooth buckles to singular creases, *Phys. Rev. Lett.* 107 (2011) 018103, <https://doi.org/10.1103/PhysRevLett.107.018103>.
- [181] L. Jin, S. Cai, Z. Suo, Creases in soft tissues generated by growth, *Europhys. Lett.* 95 (2011) 64002, <https://doi.org/10.1209/0295-5075/95/64002>.
- [182] T. Tallinen, J.Y. Chung, F. Rousseau, N. Girard, L. Lefèvre, L. Mahadevan, On the growth and form of cortical convolutions, *Nat. Phys.* 12 (2016) 588–593, <https://doi.org/10.1038/nphys3632>.
- [183] Q. Wang, M. Tahir, J. Zhang, X. Zhao, Dynamic electrostatic lithography: Multiscale on-demand patterning on large-area curved surfaces, *Adv. Mater.* 24 (2012) 1947–1951, <https://doi.org/10.1002/adma.201200272>.
- [184] Q. Wang, L. Zhang, X. Zhao, Creasing to cratering instability in polymers under ultrahigh electric fields, *Phys. Rev. Lett.* 106 (2011) 118301, <https://doi.org/10.1103/PhysRevLett.106.118301>.
- [185] E. Hohlfield, Creasing, Post-bifurcations and the Spontaneous Breakdown of Scale Invariance, Harvard University, 2008.
- [186] T. Tallinen, J.S. Biggins, L. Mahadevan, Surface sulci in squeezed soft solids, *Phys. Rev. Lett.* 110 (2013) 024302, <https://doi.org/10.1103/PhysRevLett.110.024302>.
- [187] E. Hohlfield, L. Mahadevan, Unfolding the sulcus, *Phys. Rev. Lett.* 106 (2011) 105702, <https://doi.org/10.1103/PhysRevLett.106.105702>.
- [188] S. Tang, B. Gao, Z. Zhou, Q. Gu, T. Guo, Dimension-controlled formation of crease patterns on soft solids, *Soft Matter* 13 (2017) 619–626, <https://doi.org/10.1039/c6sm02013e>.
- [189] E. Hohlfield, L. Mahadevan, Scale and nature of sulcification patterns, *Phys. Rev. Lett.* 109 (2012) 025701, <https://doi.org/10.1103/PhysRevLett.109.025701>.
- [190] H. Alawiyeh, P.E. Farrell, A. Goriely, Revisiting the wrinkling of elastic bilayers II: Post-bifurcation analysis, *J. Mech. Phys. Solids* 143 (2020) 104053, <https://doi.org/10.1016/j.jmps.2020.104053>.
- [191] L. Jin, Z. Suo, Smoothening creases on surfaces of strain-stiffening materials, *J. Mech. Phys. Solids* 74 (2015) 68–79, <https://doi.org/10.1016/j.jmps.2014.10.004>.
- [192] J. Yang, L. Jin, J.W. Hutchinson, Z. Suo, Plasticity retards the formation of creases, *J. Mech. Phys. Solids* 123 (2019) 305–314, <https://doi.org/10.1016/j.jmps.2018.08.016>.
- [193] J. Yang, W. Illeperuma, Z. Suo, Inelasticity increases the critical strain for the onset of creases on hydrogels, *Extrem. Mech. Lett.* 40 (2020) 100966, <https://doi.org/10.1016/j.eml.2020.100966>.
- [194] P. Ciarletta, L. Truskinovsky, Soft nucleation of an elastic crease, *Phys. Rev. Lett.* 122 (2019) 248001, <https://doi.org/10.1103/PhysRevLett.122.248001>.
- [195] K.R. Shull, C.M. Flanagan, A.J. Crosby, Fingering instabilities of confined elastic layers in tension, *Phys. Rev. Lett.* 84 (2000) 3057–3060, <https://doi.org/10.1103/PhysRevLett.84.3057>.
- [196] S. Lin, T. Cohen, T. Zhang, H. Yuk, R. Abeyaratne, X. Zhao, Fringe instability in constrained soft elastic layers, *Soft Matter* 12 (2016) 8899–8906, <https://doi.org/10.1039/c6sm01672c>.
- [197] A. Sharma, A. Ghatak, M. Chaudhury, V. Shenoy, Meniscus instability in a thin elastic film, *Phys. Rev. Lett.* 85 (2000) 4329–4332, <http://www.ncbi.nlm.nih.gov/pubmed/11060630>.
- [198] A.N. Gent, P.B. Lindley, Internal rupture of bonded rubber cylinders in tension, *Proc. R. Soc. Lond. Ser. A. Math. Phys. Sci.* 249 (1959) 195–205, <https://doi.org/10.1098/rspa.1959.0016>.
- [199] A.N. Gent, B. Park, Failure processes in elastomers at or near a rigid spherical inclusion, *J. Mater. Sci.* 19 (1984) 1947–1956.
- [200] C. Fond, Cavitation criterion for rubber materials: A review of void-growth models, *J. Polym. Sci. Part B Polym. Phys.* 39 (2001) 2081–2096, <https://doi.org/10.1002/polb.1183>.
- [201] C. Creton, M. Ciccotti, Fracture and adhesion of soft materials: A review, *Rep. Prog. Phys.* 79 (2016) 46601, <https://doi.org/10.1088/0034-4885/79/4/046601>.
- [202] C.W. Barney, C.E. Dougan, K.R. McLeod, A. Kazemi-Moridani, Y. Zheng, Z. Ye, S. Tiwari, I. Sacligil, R.A. Riggall, S. Cai, J.H. Lee, S.R. Peyton, G.N. Tew, A. J. Crosby, Cavitation in soft matter, *Proc. Natl. Acad. Sci. U. S. A.* 117 (2020) 9157–9165, <https://doi.org/10.1073/pnas.1920168117>.
- [203] J.S. Biggins, L. Mahadevan, Meniscus instabilities in thin elastic layers, *Soft Matter* 14 (2018) 7680–7689, <https://doi.org/10.1039/c8sm01033a>.
- [204] J.S. Biggins, B. Saintyves, Z. Wei, E. Bouchaud, L. Mahadevan, Digital instability of a confined elastic meniscus, *Proc. Natl. Acad. Sci. U. S. A.* 110 (2013) 12545–12548, <https://doi.org/10.1073/pnas.1302269110>.
- [205] B. Giovannardi, A.A. Sliwiak, A. Koshakji, S. Lin, X. Zhao, R. Radovitzky, A path-following simulation-based study of elastic instabilities in nearly-incompressible confined cylinders under tension, *J. Mech. Phys. Solids* 131 (2019) 252–275, <https://doi.org/10.1016/j.jmps.2019.06.020>.
- [206] S. Lin, Y. Mao, R. Radovitzky, X. Zhao, Instabilities in confined elastic layers under tension: Fringe, fingering and cavitation, *J. Mech. Phys. Solids* 106 (2017) 229–256, <https://doi.org/10.1016/j.jmps.2017.05.011>.

- [207] S. Lin, Y. Mao, H. Yuk, X. Zhao, Material-stiffening suppresses elastic fingering and fringe instabilities, *Int. J. Solids Struct.* 139–140 (2018) 96–104, <https://doi.org/10.1016/j.ijsolstr.2018.01.022>.
- [208] T. Vilmin, F. Ziebert, E. Raphaël, Simple view on fingering instability of debonding soft elastic adhesives, *Langmuir* 26 (2010) 3257–3260, <https://doi.org/10.1021/la903013z>.
- [209] B. Saintyves, O. Dauchot, E. Bouchaud, Bulk elastic fingering instability in Hele-Shaw cells, *Phys. Rev. Lett.* 111 (2013) 047801, <https://doi.org/10.1103/PhysRevLett.111.047801>.
- [210] A. Ghatak, L. Mahadevan, J.Y. Chung, M.K. Chaudhury, V. Shenoy, Peeling from a biomimetically patterned thin elastic film, *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* 460 (2004) 2725–2735, <https://doi.org/10.1098/rspa.2004.1313>.
- [211] J.Y. Chung, K.H. Kim, M.K. Chaudhury, J. Sarkar, A. Sharma, Confinement-induced instability and adhesive failure between dissimilar thin elastic films, *Eur. Phys. J. E.* 20 (2006) 47–53, <https://doi.org/10.1140/epje/i2005-10080-0>.
- [212] M. Ben Amar, D. Bonn, Fingering instabilities in adhesive failure, *Phys. D Nonlinear Phenom.* 209 (2005) 1–16, <https://doi.org/10.1016/j.physd.2005.07.002>.
- [213] J.Y. Chung, M.K. Chaudhury, Roles of discontinuities in bio-inspired adhesive pads, *J. R. Soc. Interface.* 2 (2005) 55–61, <https://doi.org/10.1098/rsif.2004.0020>.
- [214] A. Ghatak, M.K. Chaudhury, Adhesion-induced instability patterns in thin confined elastic film, *Langmuir* 19 (2003) 2621–2631, <https://doi.org/10.1021/la026932t>.
- [215] P.G. Saffman, G.I. Taylor, The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid, *Proc. R. Soc. Lond. Ser. A Math. Phys. Sci.* 245 (1958) 312–329, <https://doi.org/10.1098/rspa.1958.0085>.
- [216] M.H.B.M. Shariff, R. Bustamante, J. Merodio, A nonlinear constitutive model for a two preferred direction electro-elastic body with residual stresses, *Int. J. Non. Linear. Mech.* 119 (2020) 103352, <https://doi.org/10.1016/j.ijnonlinmec.2019.103352>.
- [217] M.H.B.M. Shariff, J. Merodio, Residually stressed two fibre solids: A spectral approach, *Int. J. Eng. Sci.* 148 (2020) 103205, <https://doi.org/10.1016/j.ijengsci.2019.103205>.
- [218] R.H.W. Funk, T. Monsees, N. Özküçür, Electromagnetic effects – From cell biology to medicine, *Prog. Histochem. Cytochem.* 43 (2009) 177–264, <https://doi.org/10.1016/j.proghi.2008.07.001>.
- [219] M. Levin, Bioelectric mechanisms in regeneration: Unique aspects and future perspectives, *Semin. Cell Dev. Biol.* 20 (2009) 543–556, <https://doi.org/10.1016/j.semcdb.2009.04.013>.